

Neutrino mixing and masses in SO(10) GUTs with hidden sector and flavor symmetries

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Abstract

We consider the neutrino masses and mixing in the framework of SO(10) GUTs with hidden sector consisting of fermionic and bosonic SO(10) singlets and flavor symmetries. The framework allows to disentangle the *CKM physics* responsible for the CKM mixing and different mass hierarchies of quarks and leptons and the *neutrino new physics* which produces smallness of neutrino masses and large lepton mixing. The framework leads naturally to the relation $U_{PMNS} \sim V_{CKM}^\dagger U_0$, where structure of U_0 is determined by the flavor symmetry. The key feature of the framework is that apart from the Dirac mass matrices m_D , the portal mass matrix M_D and the mass matrix of singlets M_S are also involved in generation of the lepton mixing. This opens up new possibilities to realize the flavor symmetries and explain the data. Using $A_4 \times Z_4$ as the flavor group, we systematically explore the flavor structures which can be obtained in this framework depending on field content and symmetry assignments. We formulate additional conditions which lead to $U_0 \sim U_{TBM}$ or U_{BM} . They include (i) equality (in general, proportionality) of the singlet flavons couplings, (ii) equality of their VEVs; (iii) correlation between VEVs of singlets and triplet, (iv) certain VEV alignment of flavon triplet(s). These features can follow from additional symmetries or be remnants of further unification. Phenomenologically viable schemes with minimal flavon content and minimal number of couplings are constructed.

1 Introduction

There are several hints in favor of a framework with Grand unification, hidden sector at the string/Planck scale, and flavor symmetries acting both in visible and hidden sectors:

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1. Grand Unification Theories (GUT) are still one of the highly motivated and very appealing scenarios of physics beyond the Standard Model (SM) [1, 2]. In particular, the GUT models based on $SO(10)$ [3, 4] beside the gauge coupling unification and charge quantization accommodate exactly the quantum numbers of all known SM fermions, plus right-handed (RH) neutrinos. This can not be accidental. The $SO(10)$ GUT also provides natural realizations of the high scale seesaw mechanism [5–9], and therefore explanation of the smallness of neutrino masses.

2. In fact, the required mass scale of the RH neutrinos from the seesaw mechanism, $M_N \sim m_D^2/m_\nu \sim (10^8 - 10^{14})$ GeV, where m_D is a typical Dirac mass of the quarks and charged leptons, is somehow below the GUT scale. Nevertheless, the scale of M_N can be made of the GUT scale M_{GUT} via the seesaw mechanism, if additional fermion singlets S exist with masses at the Planck scale M_{Pl} : $M_N \sim M_{GUT}^2/M_{Pl}$. Consequently, the double seesaw mechanism is realized for light neutrino masses [10].

3. Generically, the unification is at odd with strong difference of mixings and masses of quarks and leptons. Partially, the difference of mixings can be related to the smallness of neutrino mass [11–13] and the latter can originate again from the seesaw mechanism.

4. The approximate tri-bimaximal (TBM) lepton mixing [14] can be considered as an indication of existence of the flavor symmetry behind [15–17]. Furthermore, simple discrete groups, like A_4 [18, 19], have certain properties which correspond to features of the TBM mixing [20–25]. In $SO(10)$, realization of such a symmetry faces serious problems in view of strongly different mixing pattern in the quark sector [26, 27]. For some realizations with A_4 in $SO(10)$ GUT, see e.g. [28–30].

5. Although being strongly different, the quark and lepton mixings can be related. Indeed, experimental data confirm the following connection between the quark, V_{CKM} , and lepton, U_{PMNS} , mixing matrices [31–39]

$$U_{PMNS} \simeq V_{CKM}^\dagger U_0. \quad (1)$$

Here U_0 is the matrix of special form, like the TBM mixing matrix U_{TBM} or the bi-maximal (BM) mixing U_{BM} [40–44], dictated by symmetry and related to some new structures of theory which affect the lepton sector only. Eq. (1) gives a relation between the 1-3 mixing, 2-3 mixing and CKM mixing, which is in a good agreement with data [45]. In the lowest-order (only Cabibbo angle θ_C in V_{CKM} , and maximal 2-3 mixing) the relation reduces to $\sin^2 \theta_{13} = 0.5 \sin^2 \theta_C$ [32, 46].

The relation (1) implies Grand unification, which transfers information about mixing from the quark to lepton sector.

6. The double seesaw mechanism allows to explain difference of mixing patterns in the quark and lepton sectors and reproduce the relation (1). It reveals new possibilities to realize flavor symmetries. Striking differences in the quark and lepton sectors follow essentially from the mixing of new fermion singlets S with neutrino (left- and right-handed) components of 16-plet, thus providing the main source of the lepton mixing.

Origin of singlets S can be related to further extension of the gauge symmetry: $SO(10) \rightarrow E_6$, in which new singlets are additional neutral components of 27-plets [47]. Alternatively, singlet fermions S can be components of the hidden sector composed of the fermionic and bosonic singlets of $SO(10)$. Hidden sector may have its own gauge symmetries. Sterile neutrinos, if exist, particles of dark matter, axions, *etc.*, can also originate from this sector.

It seems that the existing data on masses and mixing are too complicated to be explained all at once, from a few principles and a few parameters. So, according to Eq. (1), probably two different (but somehow connected) types of new physics are involved: the “CKM new physics” responsible for the CKM mixing and difference of hierarchies of Dirac masses, and the “neutrino new physics”, which explains smallness of neutrino masses and large mixing with its form dictated by symmetry. The neutrino new physics is related to the hidden sector and properties of the portal interactions which connect the visible sector and the hidden one [48–50].

This framework has been explored to some extent previously. That includes (i) the seesaw enhancement of lepton mixing due to coupling with new singlets [11], (ii) screening of the Dirac structures, which means that the mixing of light neutrinos is essentially related to mixing in the singlet sector S : $m_\nu \propto M_S$ [11, 51], (iii) stability of this screening with respect to renormalization group (RG) running [51], renormalization of the relation (1) [52], (iv) symmetry origins (groups T_7 and Σ_{81} have been used) of the screening, as well as partial screening [53], (v) lepton mixing from the symmetries of the hidden sector [45].

In this paper we further elaborate on the framework with emphasis on new mechanisms of getting flavor structures from symmetries. We systematically explore the fermion masses and mixing matrices which can be obtained in the framework. Interplay of the flavor and GUT symmetries is crucial. We mainly discuss here the symmetry aspects and formulate conditions of the vacuum alignment which eventually should be realized in complete model buildings.

The paper is organized as follows. In sect. 2 we study properties of the mass matrices involved in generation of the neutrino masses and mixing in this framework. We elaborate on the main elements and building blocks of the schemes of lepton mixing. In sect. 3 we present the simplest viable schemes with fermion singlets S transforming under three-dimensional representation $\mathbf{3}$ of A_4 . In sect. 4 the schemes are considered with S transforming under one-dimensional representations $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ of A_4 . Sect. 5 is devoted to various generalizations: effect of linear seesaw, contribution from additional singlets, elements of the CKM physics. We summarize our results in sect. 6. Some technical details are presented in the Appendix.

2 Neutrino mass and mixing matrices

2.1 The framework

Let us first summarize main ingredients of the framework outlined in the introduction. We consider the $SO(10)$ gauge model with three generations of fermions and the following additional features:

- All the chiral fermions (including the right-handed neutrino) of each generation are accommodated in a 16-dimensional spinor representation, 16_{Fi} ($i = 1, 2, 3$).
- Three or more singlet fermions S_i exist. They mix with neutrinos due to Yukawa interactions with 16-plets of Higgs scalars $\overline{16}_H$.
- Only low-dimension representations of Higgs fields, 10_H , $\overline{16}_H$ and 1_H , give masses to fermions. High dimensional representations 126_H and 120_H are absent.

- Approximate flavor symmetry exists with fermions 16_{Fi} , S_j carrying non-zero flavor charges. For definiteness we consider the simplest discrete group which has irreducible three-dimensional representations, A_4 . Various features obtained from this symmetry can be also reproduced with other groups.
- Three generations of the matter fields, 16_{Fi} , transform as triplet of A_4 in most of the cases. We explore different symmetry assignments for three singlets S_i , $(\mathbf{3}, \mathbf{1}, \mathbf{1}', \mathbf{1}'')$.
- A number of flavons (scalar fields, $SO(10)$ singlets) exist, which have non-trivial transformations with respect to A_4 and acquire vacuum expectation values (VEV), thus breaking the symmetry.
- The Higgs multiplets, 10_H and 16_H , are singlets of A_4 , and therefore factor out of the flavor structure in the simplest cases. This reduces the complexity in obtaining certain form of mass matrices and mixing from flavor symmetries. Still, 10_H and 16_H may carry charges of additional symmetries, in which case they will affect the flavor.

We do not specify here the $SO(10)$ symmetry breaking, although the corresponding Higgs multiplets can also take part in generation of the fermion masses. Rather than constructing complete models, we will explore systematically the flavor structures which can be obtained in the formulated framework.

2.2 Yukawa couplings and mass matrices

The Lagrangian has three types of Yukawa couplings,

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_{NS} + \mathcal{L}_S, \quad (2)$$

which produce masses of fermions:

- 1). The *visible* (Dirac) sector Yukawa interactions

$$\mathcal{L}_D = Y_{10}^{ija} \cdot 16_{Fi} 16_{Fj} 10_H^a \quad (3)$$

generate after symmetry breaking the usual Dirac masses of fermions. With just one 10_H these couplings produce Dirac mass matrices of the up-type quark, down-type quark, charge lepton mass matrices and neutrinos of the same form:

$$m_u = m_D = Y_{10} v_u, \quad m_d = m_l = Y_{10} v_d, \quad (4)$$

where v_u v_d are the VEVs of 10_H . No quark mixing is generated at this level. Nevertheless, such a mixing can be produced if two (or more) different 10_H^a ($a = u, d$) are introduced (see sect. 5.3).

- 2). The *portal* interactions,

$$\mathcal{L}_{NS} = Y_{16}^{ij} \cdot 16_{Fi} S_j \overline{16}_H, \quad (5)$$

generate mass matrices of Dirac type for the LH and RH neutrinos from 16_{Fi} and singlets S_j

$$m' = Y_{16} v_L, \quad M_D = Y_{16} v_R.$$

Here v_L, v_R are the VEVs of the $SU(2)_L$ and $SU(2)_R$ doublets from $\overline{16}_H$. As long as only one Higgs 16-plet generates M_D and m' , the two mass matrices are proportional to each other:

$$m' = \frac{v_L}{v_R} M_D. \quad (6)$$

3). The *hidden sector* interactions,

$$\mathcal{L}_S = Y_1^{ij} \cdot S_i S_j 1_H + M_S^{0ij} S_i S_j, \quad (7)$$

produce the mass matrix of singlets

$$M_S = Y_1 \langle 1_H \rangle + M_S^0.$$

Here 1_H fields are singlets under both flavon symmetry and $SO(10)$, which may not be necessary and are assumed to be absorbed into flavons from now on. The masses of singlets can be generated at the renormalizable level, and also bare mass terms M_S^0 can appear for some or all singlets if it is not forbidden by flavor symmetry.

Notice that here the Yukawa couplings are effective couplings – functions of VEVs of flavon fields, Φ :

$$Y_k^{ija} = Y_k^{ija}(\langle \Phi \rangle).$$

If $\langle \Phi \rangle \ll \Lambda_f$, where Λ_f is a scale of flavor physics, the constants can be expanded as

$$Y_k^{ija} = y_k^{ija} + \left[y_k \frac{\langle \Phi \rangle}{\Lambda_f} \right]^{ija} + \left[y_k^2 \frac{\langle \Phi \rangle^2}{\Lambda_f^2} \right]^{ija} + \dots, \quad k = 10, 16, 1.$$

In what follows we will take the lowest-order approximation in $\langle \Phi \rangle / \Lambda_f$.

The mass matrix of neutral leptons generated by interactions (3), (5) and (7) in the basis (ν, N, S) , where $N \equiv \nu^c$, equals

$$\begin{pmatrix} 0 & m_D & m' \\ m_D^T & 0 & M_D \\ m'^T & M_D^T & M_S \end{pmatrix}. \quad (8)$$

Given the Yukawa interactions introduced above, the matrices m_D and M_S are symmetric: $m_D = m_D^T$ and $M_S = M_S^T$, while m' and M_D may have a non-symmetric form. The mass terms are defined as $\nu^T m_D C^{-1} N$, $N^T M_D C^{-1} S$, etc., and $C \equiv i\gamma^0 \gamma^2$. For

$$m_D, m' \ll M_D, M_S$$

the diagonalization of (8) leads to the light neutrino mass matrix

$$m_\nu \simeq m_D \frac{1}{M_D^T} M_S \frac{1}{M_D} m_D^T - \left(m_D \frac{1}{M_D^T} m'^T + m' \frac{1}{M_D} m_D^T \right), \quad (9)$$

where the first term is the double seesaw contribution, and two others correspond to the linear seesaw contribution. In the case of a single $\overline{16}_H$, according to Eq. (6) the mass matrix (9) becomes

$$m_\nu \simeq m_D \frac{1}{M_D^T} M_S \frac{1}{M_D} m_D^T - \frac{2v_L}{v_R} m_D. \quad (10)$$

Given that $m_\nu \sim \mathcal{O}(0.1) \text{ eV}$ and $v_L < 246 \text{ GeV}$, it is possible to define a critical mass scale

$$M_{cr} = 4 \times 10^{14} \left(\frac{v_L \cdot Y_{16}}{246 \text{ GeV}} \right)^2 \text{ GeV}, \quad (11)$$

so that for $M_S \ll M_{cr}$ the "linear-seesaw" dominates, whereas for $M_S \gg M_{cr}$, the "double-seesaw" gives the main contribution. In our framework with $M_S \sim M_{Pl}$ the "double-seesaw" dominates and the "linear-seesaw" term may lead to small corrections.

According to Eq. (4) and $m_D \propto m_l$, no mixing is produced by the Dirac mass matrices. In contrast, due to extended structure of the mass matrix of neutral leptons Eq. (8), m_ν differs from m_l , and so the lepton mixing is generated. In the basis where the Dirac mass matrices are diagonal, $m_D = m_D^{diag}$, the origin of lepton mixing is the structure

$$m_D^{diag} M_D^{-1T} M_S M_D^{-1} m_D^{diag}. \quad (12)$$

In the lowest order, the Majorana mass term of right-handed neutrinos, $N^T C^{-1} N$, is absent, since we do not introduce 126 Higgs multiplet. Such a term can be generated by higher-dimension operator $(1/\Lambda) 16_{Fi} 16_{Fj} (16_H 16_H)^{ij}$, or effectively generated by the decoupling of heavy particles with mass $\sim \Lambda$. It then gives $M_N = v_R^2/\Lambda$.

In our framework, the mass term $N^T C^{-1} N$ appears after integrating out S , which gives

$$M_N = M_D^T M_S^{-1} M_D = v_R^2 Y_{16}^T M_S^{-1} Y_{16}. \quad (13)$$

We assume that this contribution to M_N dominates. Other contributions either are forbidden by certain auxiliary symmetry or have the same flavor structure as Eq. (13).

Notice that since in our framework there is $m_D \ll M_D$, the RG running of Yukawa couplings may modify the expressions for neutrino masses non-negligibly, and thus may need to be taken into account [54–58].

2.3 Screening and partial screening

The double seesaw allows to realize the screening of the Dirac structures [11, 51], which will be used in many examples presented in this paper. If

$$m_D \propto M_D^T, \quad (14)$$

then according to Eq. (10)

$$m_\nu \propto M_S, \quad (15)$$

i.e. structure of the light neutrino mass matrix is given by that of M_S , thus disentangling properties of neutrinos and quarks completely. The form of M_S can strongly differ from those of m_D and M_D , thus leading to very different lepton mixing.

Suppose that the screening condition is fulfilled in the basis where $m_D \propto M_D^T$ are diagonal due to some certain symmetry, *i.e.* in the flavor basis, and that in this basis the singlet mass matrix is M_S . Then according to (15) the mixing is determined exclusively by M_S .

If the screening condition (14) is fulfilled in some other basis (again determined by symmetry) where $m_l = m_D$ is non-diagonal and the singlet matrix is M'_S , then $m_\nu \propto M'_S$. But

now the lepton mixing gets contributions both from m_D and M'_S . We present schemes which realize both possibilities.

Let us consider the screening condition in more details. We define the unitary transformations U_l , V_R , V_S and V_N which diagonalize the mass matrices m_D and M_D , so that

$$m_D = U_l m_D^{diag} V_R^\dagger, \quad M_D^T = V_S M_D^{diag} V_N^\dagger.$$

Then the screening matrix (factor) equals

$$m_D (M_D^T)^{-1} = U_l m_D^{diag} V_R^\dagger V_N (M_D^{diag})^{-1} V_S^\dagger.$$

The screening (14) implies that

$$V_R = V_N, \quad m_D^{diag} \propto M_D^{diag}, \quad U_l = V_S.$$

If only the first condition (the same rotations of the RH neutrino components) and the second one are fulfilled we obtain

$$m_D (M_D^T)^{-1} = U_l V_S^\dagger \neq I. \quad (16)$$

This partial screening gives additional contribution to the neutrino mixing. Due to symmetry the matrices U_l and V_S can be related in such a way that Eq. (16) produces a matrix of special form, which then leads to the required form of total mixing matrix (see sect. 3.5). The screening can be due to further embedding of singlets S in extended gauge symmetry or due to common flavor symmetries.

Some additional physics (which we call the CKM physics) can lead to the misalignment of mass matrices for up- and down- type quarks. Due to GUT similar misalignment is expected between the neutrino Dirac matrix m_D and the charged lepton matrix $m_l \neq m_D$. Then in the presence of screening $m_D \propto M_D^T = diag$ the PMNS mixing gets the CKM type contribution $\sim V_{CKM}$ from m_l and another one, U_S , from M_S :

$$U_{PMNS} = V_{CKM}^\dagger U_S.$$

This allows to maximally disentangle the CKM and neutrino new physics and realize the scenario in Eq. (1).

2.4 Symmetry and flavor structures

The flavor symmetry imposes restrictions on the Yukawa couplings Y_k^{ij} , and thus forbids some of them or leads to relations between them. For definiteness we use A_4 flavor group (see Appendix. A for details). Various structures we obtain in this paper can be reproduced in models with other symmetry groups which include representations **3**, **1**, **1'**, **1''**. On the other hand, with additional symmetries it is possible to produce in our framework features from other symmetry groups.

We introduce flavon triplets ξ and singlets $\vec{\varphi} = (\varphi, \varphi', \varphi'')$, which acquire the VEV $\langle \xi \rangle \equiv (u_1, u_2, u_3)$ and $(\langle \varphi \rangle, \langle \varphi' \rangle, \langle \varphi'' \rangle) \equiv (v, v', v'')$. The general form of a fermion mass term is $F_i F_j$, where the fermionic multiplets $F_i = 16_F$, S . Depending on symmetry assignment

of F_i and for general sets of the flavon fields, which include the flavon triplets ξ and singlets $\vec{\varphi}$, we obtain three types of mass matrices:

1. $F_1 \sim \mathbf{3}, F_2 \sim \mathbf{3}$. The most general coupling with flavons invariant under A_4 is

$$m_0(F_1 F_2) + \vec{y}(F_1 F_2)_\varphi \vec{\varphi} + h(F_1 F_2)\xi, \quad (17)$$

where we use notations

$$\vec{y}(F_1 F_2)_\varphi \vec{\varphi} \equiv y(F_1 F_2)\varphi + y'(F_1 F_2)''\varphi' + y''(F_1 F_2)'\varphi''. \quad (18)$$

Here $\vec{y} \equiv (y, y', y'')$ and $(Q)_\varphi$ ($Q = F_1 F_2$ in this case) is the field operator which produces one-dimensional representations of A_4 together with a given φ .

The mass term $m_0(F_1 F_2)$ without flavons can be forbidden by symmetries. The interactions (17) lead to the mass terms $F_1 M_{33} F_2^T + h.c.$ with

$$M_{33} = M_{33}^{diag} + M_{33}^{off} = m_0 \mathbb{1} + M_{33}^\varphi + M_{33}^\xi, \quad (19)$$

where the off-diagonal part is generated by flavon triplet, whereas the diagonal one

$$M_{33}^{diag} = m_0 \mathbb{1} + M_{33}^\varphi$$

is due to coupling with the flavon singlets of A_4 and flavonless contribution. Three terms in (19) are generated by different flavons. Depending on additional symmetry assignments one, or two, or all three terms can contribute. Explicitly,

$$M_{33}^\varphi = \begin{pmatrix} yv + y'v' + y''v'' & 0 & 0 \\ 0 & yv + \omega y'v' + \omega^2 y''v'' & 0 \\ 0 & 0 & yv + \omega^2 y'v' + \omega y''v'' \end{pmatrix}. \quad (20)$$

Notice that for all equal values ($yv = y'v' = y''v''$), $M_{33}^{diag} = \text{diag}(m_0 + 3yv, m_0, m_0)$, which can be a good first approximation in some cases. For real parameters $\mu \equiv m_0 + yv$, $\mu' \equiv y'v'$ and $\mu'' \equiv y''v''$ two eigenstates are degenerate:

$$|M_2| = |M_3| = [(\mu + \mu' + \mu'')^2 - 3(\mu\mu' + \mu'\mu'' + \mu\mu'')]^{1/2}.$$

This important feature together with certain forms of the off-diagonal part can lead to maximal mixing. General situation is analyzed in the Appendix. B.

In the absence of the bare mass m_0 , the eigenvalues of matrix (20) can be written as

$$(M_1, M_2, M_3)^T = \sqrt{3}U_\omega(yv, y'v', y''v'')^T \quad (21)$$

where

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \quad (22)$$

is the magic matrix. The matrix U_ω is unitary, $U_\omega U_\omega^\dagger = I$ and the inverse $U_\omega^{-1} = U_\omega^\dagger = U_\omega^*$, can be obtained by substitution $\omega \leftrightarrow \omega^2$.

Then the condition

$$yv \approx \omega^2 y' v' \approx \omega y'' v'' \quad (23)$$

leads to the hierarchy of masses:

$$m_3 = 3yv \gg m_2, m_1.$$

The hierarchy between the lighter states, $m_2 \gg m_1$, can be obtained by special tuning of relations in Eq. (23) or introduction of other contributions, e.g. from the linear seesaw terms.

In general, it is difficult to get correlation (23) between VEVs and Yukawa couplings. The simplest realization would be two separate equalities:

$$y \approx y' \approx y'',$$

as a result of additional symmetry, or the fact that singlets originate from breaking of representation **3** and

$$v \approx \omega^2 v' \approx \omega v'',$$

as a consequence of special forms of the potential.

In Eq. (19) the off-diagonal matrix equals

$$M_{33}^\xi = h \begin{pmatrix} 0 & u_3 & u_2 \\ \dots & 0 & u_1 \\ \dots & \dots & 0 \end{pmatrix}. \quad (24)$$

It has the form of the Zee-Wolfenstein mass matrix [59, 60] with the eigenvalues satisfying the equality $m_1 + m_2 + m_3 = 0$. For arbitrary values of u_i , the diagonalization matrix and eigenvalues of M_{33}^ξ are presented in Appendix. C, and here we consider some special cases.

If $u_1 = u_2$, the matrix can be diagonalized by maximal 2-3 rotation and vanishing θ_{13} . Then for the 1-2 mixing we obtain $\sin^2 2\theta_{12} = 8u_1^2/(8u_1^2 + u_2^2)$. If all VEVs are equal, $u_1 = u_2 = u_3$, then $\sin^2 2\theta_{12} = 8/9$, which corresponds to the TBM mixing.

Let us consider a special case of the total matrix M_{33} (19), which will appear often in our constructions. If $u_1 = u_2 = u_3 \equiv u$ and also the diagonal elements are all equal, we have

$$M_{33}^{special} = \begin{pmatrix} \mu & \beta & \beta \\ \dots & \mu & \beta \\ \dots & \dots & \mu \end{pmatrix}, \quad (25)$$

where $\beta \equiv hu$ [61]. Such a matrix can be obtained when φ' and φ'' do not contribute to masses, or when the bare mass terms or flavonless operators dominate. This matrix having the TBM form is diagonalized by U_{TBM} , see Appendix. D. The eigenvalues equal

$$\lambda_1 = \lambda_3 = \mu - \beta, \quad \lambda_2 = \mu + 2\beta, \quad (26)$$

of which two are equal and the sum is given by

$$\sum_i \lambda_i = 3\mu.$$

For $\mu = 0$ we have $\sum_i \lambda_i = 0$, and therefore $M_{33}^{special}(\mu = 0)$ can not be used to describe masses of quarks or charged leptons.

Degeneracy of the two eigenstates implies that the diagonalization matrix is not unique, and actually there is infinite ambiguity in diagonalization related to rotation in the space of equal eigenstates. In particular, the matrix (25) can be *unitarily* diagonalized by the magic matrix as¹

$$U_\omega^* M_{33}^{special} U_\omega.$$

The diagonalization gives $\lambda_1 = \mu + 2\beta$ and $\lambda_2 = \lambda_3 = \mu - \beta$, which coincides with Eq. (26) up to permutation.

The ambiguity is removed when the degeneracy of eigenvalues is broken. Let us consider correction matrices of the form

$$\Delta M = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{or} \quad \Delta M' = M'_0 \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (27)$$

of which both can arise from a Z_3 symmetry, with different charge assignments. The former symmetric matrix fixes the diagonalization matrix to U_{TBM} , whereas the latter one fixes it to U_ω . Note that U_ω can only be obtained from non-symmetric corrections, such as $\Delta M'$.

It is easy to get maximal mixing from M_{33} , which will be often needed for our constructions below. For instance, the maximal 2-3 mixing can be obtained if $\langle \xi \rangle = (u_1, 0, 0)$ and $y'v' = y''v''$, or $\vec{y} = 0$. For maximal 1-3 mixing one needs $\langle \xi \rangle = (0, u_2, 0)$ and $y'v' = \omega y''v''$.

In general, by varying yv , $y'v'$, $y''v''$ as well as hu_i , one can obtain an arbitrary symmetric matrix for M_{33} .

2. $F_1 \sim \mathbf{3}$ and $\vec{F}_2 \equiv (F_2, F'_2, F''_2) \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. The most general couplings with flavons

$$\vec{h} (F_1 \xi)_{F_2} \vec{F}_2 \equiv h(F_1 \xi) F_2 + h'(F_1 \xi)' F'_2 + h''(F_1 \xi)'' F''_2 \quad (28)$$

generate the mass term $F_1 M_{31} F_2^T$ with

$$M_{31} = \begin{pmatrix} hu_1 & h'u_1 & h''u_1 \\ hu_2 & h'u_2\omega & h''u_2\omega^2 \\ hu_3 & h'u_3\omega^2 & h''u_3\omega \end{pmatrix}. \quad (29)$$

Obviously, only flavon triplets contribute to M_{31} [62, 63]. This matrix can be represented in the form

$$M_{31} = \sqrt{3} D_u U_\omega D_h, \quad (30)$$

where $D_u \equiv \text{diag}(u_1, u_2, u_3)$ and $D_h \equiv \text{diag}(h, h', h'')$.

¹ Note that in terms of the SM components, the Dirac mass terms read

$$(l_L^c)^T C m_D l_L + l_L^T m_D C l_L^c + h.c.,$$

where l_L and $(l^c)_L$ are the left and the right components, and m_D is a symmetric matrix. Then the unitary diagonalization

$$U_\omega^* m_D U_\omega = \text{diag}(m_1, m_2, m_3),$$

would imply that l_L and $(l^c)_L$ transform differently, but l_L and l_R transform in the same way.

Additional symmetries related to transformations of components of \vec{F}_2 may lead, e.g. to $h = h' = h''$, thus further reducing the number of free parameters in D_h . If two or all three fermions in \vec{F}_2 have the same symmetry assignments, including charges under A_4 , e.g. $\vec{F}_2 \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, we would obtain a singular mass matrix M_{31} with two or three columns proportional to each other.

In general, the matrix (29) is non-symmetric. It can be made symmetric if $D_h \propto D_u$ (or equivalently, $u_i \propto h^i$), which can be obtained when h^i themselves are given by VEVs of some new fields.

3. $\vec{F}_1 \equiv (F_1, F'_1, F''_1) \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\vec{F}_2 \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. For the same set of the fermionic singlets, *i.e.* $\vec{F}_2 = \vec{F}_1$, the most general coupling with flavons $\varphi, \varphi', \varphi''$ is given by

$$\begin{aligned} & m_0 F_1 F_1 + 2\tilde{m}_0 F'_1 F''_1 + (y_{11} F_1 F_1 + 2y_{23} F'_1 F''_1) \varphi + (y_{22} F'_1 F'_1 + 2y_{13} F_1 F''_1) \varphi' \\ & + (2y_{12} F_1 F'_1 + y_{33} F''_1 F''_1) \varphi'' + h.c. \end{aligned} \quad (31)$$

It leads to the mass terms $\vec{F}_1 M_{11} \vec{F}_1^T$, where

$$M_{11} = \begin{pmatrix} m_0 + y_{11}v & y_{12}v'' & y_{13}v' \\ \dots & y_{22}v' & \tilde{m}_0 + y_{23}v \\ \dots & \dots & y_{33}v'' \end{pmatrix}. \quad (32)$$

For all free parameters, M_{11} is an arbitrary symmetric matrix. One can get from Eq. (32) maximal mixing or the TBM mixing immediately under the conditions

$$y_{22}v' = y_{33}v'', \quad y_{12}y_{22} = y_{13}y_{33}, \quad (m_0 - \tilde{m}_0) + (y_{11} - y_{23})v + (y_{12} - y_{33})v'' = 0. \quad (33)$$

They are satisfied if, e.g. $m_0 = \tilde{m}_0$, while all the couplings and all the VEVs are equal, respectively.

If the bare mass term dominates, it produces the structure with maximal 2-3 mixing which can be used to construct the TBM or BM mixing.

If all the couplings in Eq. (32) are equal and the bare mass terms are absent being forbidden by additional symmetry, the matrix M_{11} acquires a form

$$M_\omega = \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix}, \quad (34)$$

where $a = hv$, $b = hv'$ and $c = hv''$. The matrix M_ω is diagonalized by the magic matrix U_ω as $U_\omega^T M_\omega U_\omega$. The eigenvalues of M_ω in this case equal

$$(m_1, m_2, m_3)^T = \sqrt{3} U_\omega^* (a, b, c)^T,$$

and there is no mass (eigenvalue) degeneracy provided that none of α , $\omega\alpha$, and $\omega^2\alpha$ is real, where $\alpha \equiv ab^* + bc^* + ca^*$. Notice that Eq. (34) is the only form of symmetric matrix which produces the mixing $U_l = U_\omega^{*2}$.

²If m_D is non-symmetric, which can happen in the presence of additional 120_H or 45_H Higgs multiplets, other forms are allowed to generate U_ω .

Thus, A_4 with additional conditions (*i.e.* equalities of couplings and VEVs, which in turn imply some additional symmetries) allows to get U_{TBM} or U_ω . The latter appears from various interactions, and together with additional maximal 1-3 rotation, can again lead to U_{TBM} . In this sense A_4 has features which are close to the TBM mixing.

For $16_F \sim \mathbf{3}$, the Dirac mass matrices are of M_{33} form: $m_D = M_{33}^{(D)}$. With certain restrictions they can be further of M_{33}^φ or M_{33}^ξ type. If $S \sim \mathbf{3}$, all other matrices also have the form M_{33} : $M_D = M_{33}^{(P)}$, $M_S = M_{33}^{(S)}$ although values of elements of these matrices being functions of different couplings (y, h) and VEVs (v, u), are different. This is marked by different superscripts. Here also additional restrictions can select M_{33}^{diag} or M_{33}^ξ . All these matrices are symmetric. The mass matrix of light neutrinos equals

$$m_\nu = M_{33}^{(D)} \left[M_{33}^{(P)T} \right]^{-1} \cdot M_{33}^{(S)} \cdot \left[M_{33}^{(P)} \right]^{-1} M_{33}^{(D)}. \quad (35)$$

For $S \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ the portal and singlet mass matrices have different structures: $M_D = M_{31}^{(P)}$ and $M_S = M_{11}^{(S)}$. Correspondingly, the light neutrino mass matrix is given by

$$m_\nu = M_{33}^{(D)} (M_{31}^{(P)T})^{-1} M_{11}^{(S)} \left[M_{31}^{(P)} \right]^{-1} M_{33}^{(D)},$$

and now M_D is usually not symmetric. Using representation (30) we have

$$m_\nu = \frac{1}{3} M_{33}^{(D)} D_u^{-1} \cdot U_\omega^* \cdot (D_h^{-1} M_{11}^{(S)} D_h^{-1}) \cdot U_\omega \cdot D_u^{-1} M_{33}^{(D)}. \quad (36)$$

The screening can arise from $M_{33}^{(D)} (M_{33}^{(P)T})^{-1}$ in Eq. (35), or partial screening from $M_{33}^{(D)} D_u^{-1}$ in Eq. (36), depending on the A_4 charges of fermion singlets.

Including also the possibility of $16_F \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ we obtain four different combinations of mass matrices shown in the Table. 1. In what follows we mostly focus on the first two possibilities with $16_F \sim \mathbf{3}$ and S transforming as either $\mathbf{3}$ or $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$.

16_F under A_4	S under A_4	Dirac m_D	portal M_D	singlet M_S
$\mathbf{3}$	$\mathbf{3}$	$M_{33}^{(D)}$	$M_{33}^{(P)}$	$M_{33}^{(S)}$
$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$M_{33}^{(D)}$	$M_{31}^{(P)}$	$M_{11}^{(S)}$
$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$M_{11}^{(D)}$	$M_{13}^{(P)}$	$M_{33}^{(S)}$
$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$M_{11}^{(D)}$	$M_{11}^{(P)}$	$M_{11}^{(S)}$

Table 1: Combinations of mass matrices for various A_4 assignments.

In general, any structure of m_ν can be obtained. So, additional restrictions are needed in order to produce specific flavor structures. Further structuring can be achieved if not all possible flavons are introduced when some couplings and VEVs of flavons are forbidden (zero) or are related to each other. The latter implies an additional symmetry, such as Z_2 or Z_4 . If more than three fermionic singlets S exist we can obtain more flavor structures assigning S in triplet and singlet representations of A_4 : $S \sim \mathbf{3} + (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. This extension will be considered in some details in sect. 5.2.

Finally, in the case of 16_F being singlets of A_4 , $m_D = M_{11}^{(D)}$ in the form of Eq. (32), and some new possibilities appear:

1. As we have marked already, equal Yukawa couplings (without bare masses) lead to the Dirac mass matrix of the form Eq. (34). The later is diagonalized by U_ω . Then maximal 1-3 mixing can follow from the portal and hidden sector interactions to produce the TBM mixing.

2. If S are transforming as triplets of A_4 , partial screening, *i.e.* $m_D/M_D \propto U_\omega$, can be obtained since in this case $M_{13}^{(P)} \sim M_{31}^T \sim D_h U_\omega D_u$. However, such a construction usually corresponds to a diagonal m_D , which is difficult to generate from $m_D = M_{11}$.

2.5 Factorization of mixing matrix

In general, the lepton mixing matrix can be constructed as

$$U_{PMNS} = V_0 \times U_1 \times U_2 \times \dots,$$

where different factors come from diagonalization of different mass matrices involved. In our approach we take $V_0 \sim V_{CKM}^\dagger$. This common part for the quarks and leptons is due to the SO(10) GUT and the CKM physics. The rest, $U_0 \equiv U_{PMNS^0} = U_1 \times U_2 \dots$, reproduces U_{TBM} or U_{BM} , *i.e.* matrices of special type dictated by the flavor symmetry. Here we focus on construction of U_0 from the double seesaw mechanism, and discuss its TBM or BM forms. The CKM mixing, the non-zero 1-3 mixing, as well as other deviations from the TBM mixing, are probably related to each other and have some other origin, which we refer to as the CKM new physics. In view of small CKM mixing, U_0 can be considered as the zero-order approximation of the PMNS mixing, U_{PMNS^0} .

Flavor features which lead to TBM matrix have been extensively studied before (see [64, 65] and references therein). The real TBM matrix can be factorized as

$$U_{TBM} = \text{diag}(1, \omega^2, \omega) U_\omega U_{13}^{maxT} \text{diag}(1, 1, i).$$

This factorization is not unique: in general any product of U_ω (also with permuted rows and columns) and 2-state maximal mixings, $U_\omega U_{12}^{max}$, $U_\omega U_{13}^{max}$, or $U_\omega U_{23}^{max}$, leads to the TBM matrix with permuted rows and columns up to rephasing. This, however, does not affect physics: flavor content of the eigenstates with certain eigenvalues is the same for all the cases.

It is straightforward to generalize the factorization via U_ω and U_{13}^{maxT} to

$$\tilde{U}_\omega = D_\alpha U_\omega U_A, \quad \tilde{U}_{13}^{maxT} = U_A^\dagger U_{13}^{maxT} D_\beta,$$

where U_A is arbitrary unitary matrix, and D_α is arbitrary diagonal phase matrix and the phases can be absorbed into the wave functions of the charged leptons. The matrix D_β contributes to the Majorana phases of neutrinos. In the generalized form, one may use $U_A = (U_{12}^{maxT} U_{13}^{max})^\dagger$, so that $U_{PMNS} = \tilde{U}_\omega U_{12}^{maxT}$ and one of the factors is maximal 1-2 rotation instead of 1-3 rotation. However, now $\tilde{U}_\omega = D_\alpha U_\omega U_{13}^{maxT} U_{12}^{max}$, which is not reduced to U_ω with permuted columns (rows), and so it is difficult to obtain U_ω as a result of symmetry.

Another possible factorization of U_{TBM} , without using the magic matrix U_ω directly, is $U_{23}^{max}U_{12}(\theta_{TBM})$, where $\sin^2 \theta_{TBM} = 1/3$. However, mass matrices diagonalized by $U_{12}(\theta_{TBM})$ do not naturally appear from A_4 symmetry.

According to the double seesaw, there are three mass matrices relevant for generation of the lepton mixing:

$$m_D, \quad m_D M_D^{T-1}, \quad M_S, \quad (37)$$

where the first matrix is the mass matrix of charged leptons, the second one is the screening factor and the third one is the mass matrix of singlet fermions. In general all these matrices provide contributions to the lepton mixing matrix. Different mass matrices in Eq. (37) can be responsible for generation of different factors in the TBM matrix. Let us mention some possibilities mostly related to U_ω and U_{13}^{max} , which will be realized in specific schemes in sect. 3.

1. U_ω may follow from the charged leptons, and diagonalize the matrix m_D , whereas U_{13}^{max} comes from the neutrino mass matrix, so that

$$m_\nu = \begin{pmatrix} x & 0 & z \\ 0 & y & 0 \\ z & 0 & x \end{pmatrix}. \quad (38)$$

In the $SO(10)$ framework $U_l = U_\omega^*$ implies that $m_l = m_D = M_\omega$ defined in Eq. (34).

2. $U_l = \mathbb{1}$, which means that m_D is diagonal, U_ω follows from the portal factor: $m_D M_D^{T-1} \propto U_\omega^*$, and U_{13}^{max} originates from M_S .

3. One extreme possibility is when U_{TBM} comes completely from charged leptons, $U_l = (U_{TBM})^\dagger$. The neutrino diagonalization matrix should be the unit matrix.

4. Another extreme is when the whole U_{TBM} mixing is generated by M_S (*i.e.* by the hidden sector): $m_D = \text{diag}$, $m_D^T M_D^{-1} \propto \mathbb{1}$, $M_S = M_{TBM}$. In this case $m_\nu \propto M_S$.

2.6 Additional symmetries and VEV alignments

In what follows we construct schemes for masses and mixing using mass matrices elaborated in this section as building blocks. It is easy to show that in the formulated framework without additional constraints any mass matrix of the light neutrinos can be obtained. Therefore to get certain flavor structures we will use

- minimal possible number flavon fields;
- minimal number of (different) couplings;
- the most symmetric situations when as many as possible couplings are either zero or equal. The latter implies certain extended symmetries and for definiteness we will use additional symmetry Z_4 .

Existence of different energy scales in this framework, $m_D \ll M_D \ll M_S$, allows to realize the idea that different symmetries are realized at different scales with larger symmetry at higher mass scale. Higher scale has larger symmetry, which is then broken explicitly or spontaneously at lower scale. Thus, M_S may have additional symmetry which is not realized in portal and visible interactions. Feedback of this breaking onto higher scale physics is expected to be suppressed by factors $M_D/M_S \sim 10^{-2}$. In particular, due to additional

symmetries M_S can have the TBM form. Although the symmetry is not realized at the portal scale, the corrections to TBM mixing can be of the order 10^{-2} . See also [45] for some realizations of this idea.

We formulate conditions on couplings and VEVs which should be satisfied to get the required fermion masses and mixing. We provide some hints of how the conditions can be realized and where they can originate from.

Mechanisms have been elaborated which allow to get relations between VEVs of multiplets. In particular for triplets the alignments $v(1, 1, 1)$, or $v(0, 1, 1)$, $v(0, 0, 1)$ can be obtained, as a consequence of symmetry of the potential [66–68]. Usually such potentials have several degenerate vacua, and so some mechanisms of selection should exist. Construction of models which realize the conditions is beyond the scope of this paper.

In various cases we need also correlations of VEVs of singlet and triplet flavons:

$$h(u_1, u_2, u_3)^T = \sqrt{3}U_\omega(yv, y'v', y''v'')^T. \quad (39)$$

Multiplied by U_ω^* , the relation can be inverted to:

$$(yv, y'v', y''v'')^T = hU_\omega^*(u_1, u_2, u_3)^T/\sqrt{3}.$$

For equal constants $y' = y'' = y$, it can be written as

$$(v, v', v'')^T = aU_\omega^*(u_1, u_2, u_3)^T, \quad (40)$$

where $a = h/\sqrt{3}y$.

To reach the correlation (39) we introduce a *dimensionless* auxiliary field A with the VEVs $\langle A \rangle = (a, a, a)/\sqrt{3}$. Then the equality (39) is satisfied automatically if

$$v = \langle (A \cdot \xi) \rangle, \quad v' = \langle (A \cdot \xi)' \rangle, \quad v'' = \langle (A \cdot \xi)'' \rangle. \quad (41)$$

For non-equal constants (y, y', y'') we should substitute $v' \rightarrow v'y'/y$ and $v'' \rightarrow v''y''/y$ in the left-hand sides of these equalities. In turn, the relation (41) can be induced e.g. by scalar potentials of the form

$$V(\varphi)_{\text{eff}} \propto -[\varphi - (A \cdot \xi)]^4.$$

Consequently, $A \cdot \xi$ should have the same Z_4 charges as φ .

3 Schemes with S transforming as a triplet of A_4

For single complete set of the flavons $\xi \sim \mathbf{3}$, $\vec{\varphi} \sim \vec{\mathbf{1}}$ the most general A_4 symmetric Yukawa Lagrangian (at the lowest order) is

$$\begin{aligned} \mathcal{L}_D &= \frac{10_H}{\Lambda_f} [\vec{y}_{10} (16_F \cdot 16_F)_\varphi \vec{\varphi} + h_{10} (16_F \cdot 16_F \cdot \xi)] + y_{10}^b (16_F \cdot 16_F) 10_H, \\ \mathcal{L}_{NS} &= \frac{\overline{16}_H}{\Lambda_f} [\vec{y}_{16} (16_F \cdot S)_\varphi \vec{\varphi} + h_{16} (16_F \cdot S \cdot \xi)] + y_{16}^b (16_F \cdot S) \overline{16}_H, \\ \mathcal{L}_S &= \vec{y}_1 (S \cdot S)_\varphi \vec{\varphi} + h_1 (S \cdot S \cdot \xi) + M_S^0 (S \cdot S). \end{aligned} \quad (42)$$

Here

$$\vec{y}_{10}(16_F \cdot 16_F)_\varphi \vec{\varphi} \equiv y_{10}(16_F \cdot 16_F)\varphi + y'_{10}(16_F \cdot 16_F)''\varphi' + y''_{10}(16_F \cdot 16_F)'\varphi'',$$

etc., with general definitions of these combinations given in (18). The terms \mathcal{L}_S contain renormalizable couplings. The symmetry allows also to introduce renormalizable flavonless couplings of 16_F and S . If several sets of flavons $\vec{\varphi}_a$ and ξ_b exist, for each of them one should introduce interactions similar to those in eq. (42).

All the mass matrices (m_D , M_D and M_S) generated by this Lagrangian are of the M_{33} type plus matrices proportional to unit matrix. (The latter can be forbidden by additional symmetry.) If the same flavons contribute to all three mass matrices, these matrices are not completely independent being functions of the same set of VEVs. On the other hand, sets of coupling constants for three types of interactions, $\vec{y}_i \equiv (y_i, y'_i, y''_i)$ and h_i ($i = 10, 16, 1$), can be totally different. With this one can get arbitrary matrices m_D , M_D and M_S , and consequently, arbitrary symmetric mass matrix of the light neutrinos.

In what follows we will study the situations that some interactions are forbidden by additional symmetry Z_4 . One possibility is to keep minimal number of couplings in visible and portal sectors, and to obtain all the rest from the hidden sector. Among four types of couplings in the visible sector and portal interactions, \vec{y}_{10} , h_{10} , \vec{y}_{16} and h_{16} , at most two can be zero (one for 10_H and one for $\overline{16}_H$ if we do not take into account low-dimension interaction couplings y^b). Next complication is to assume that some couplings among \vec{y}_{10} and \vec{y}_{16} are zero, which would require more symmetries. The couplings of the hidden sector \vec{y}_1 and h_1 , are also restricted by Z_4 . Thus, we have four possibilities determined by representations of flavons that contribute to the corresponding couplings/mass terms: $h_{10} = h_{16} = 0$, which we call the singlet-singlet scheme; $\vec{y}_{10} = \vec{y}_{16} = 0$ - the triplet-triplet scheme; $h_{10} = \vec{y}_{16} = 0$ - the singlet-triplet scheme; and $\vec{y}_{10} = h_{16} = 0$ - the triplet-singlet scheme. We consider these possibilities in order and then present schemes in which both singlets and triplets contribute to the same mass terms.

3.1 Singlet - singlet flavon scheme.

The scheme with $h_{10} = h_{16} = 0$ allows to realize the following scenario:

$$U_l = \mathbb{1}, \quad m_D M_D^{-1} = \mathbb{1}, \quad U_S = U_{TBM},$$

where U_l diagonalizes m_D , and U_S diagonalizes M_S . Here the TBM mixing comes from solely the hidden sector³. Only singlet flavons and possibly flavonless operators generate visible and portal masses. The triplet flavon couplings, h_{10} and h_{16} , are forbidden by additional Z_4 symmetry. We introduce flavon singlets $\vec{\varphi}$, which operate in the visible sector and portal terms. One flavon triplet ξ of A_4 act in the hidden sector. For symmetry assignment under Z_4 we use

$$\{16_F, S | 10_H, \overline{16}_H | \vec{\varphi}, \xi\} \sim \{1, 1 | i, i | -i, 1\}. \quad (43)$$

Graphic representation of the scheme is shown in Fig. 1.

³Generation of lepton mixing in the hidden sector has been considered in [45]. The difference is that here we use common non-abelian symmetry in visible and hidden sectors.

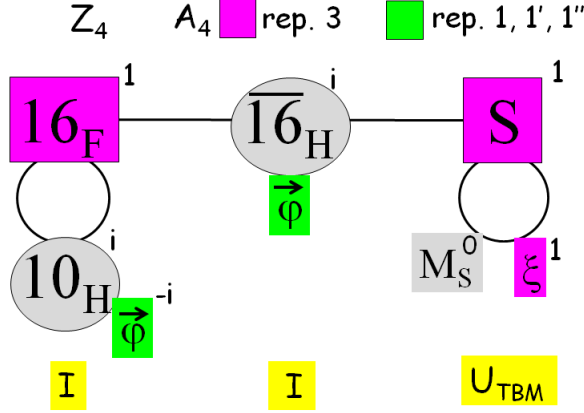


Figure 1: Graphic representation of the singlet - singlet flavon scheme. Numbers at the field notations (*i.e.* the superscript of each particle content) are the corresponding Z_4 charges. In the bottom we show rotation matrices generated by the corresponding interactions.

The most general $A_4 \times Z_4$ symmetric Lagrangian has the form

$$\begin{aligned}\mathcal{L}_D &= \frac{10_H}{\Lambda_f} \vec{y}_{10} (16_F \cdot 16_F)_\varphi \vec{\varphi}, \\ \mathcal{L}_{NS} &= \frac{\overline{16}_H}{\Lambda_f} \vec{y}_{16} (16_F \cdot S)_\varphi \vec{\varphi}, \\ \mathcal{L}_S &= M_S^0 (S \cdot S) + h_1 (S \cdot S \cdot \xi).\end{aligned}$$

All the mass matrices generated by this Lagrangian are of M_{33} type. Since triplet flavons do not appear in the visible and portal sectors, m_D and M_D are of M_{33}^{diag} type (20) leading to $U_l = \mathbb{1}$. Three eigenvalues of them are arbitrary.

For M_D we have an expression similar to m_D with substitutions $\vec{y}_{10} \leftrightarrow \vec{y}_{16}$ and $10_H \leftrightarrow \overline{16}_H$. Complete screening requires proportionality (correlation) of the couplings constants of the Dirac and portal interactions, \vec{y}_{10} and \vec{y}_{16} :

$$y_{10} : y'_{10} : y''_{10} = y_{16} : y'_{16} : y''_{16}. \quad (44)$$

This is enough for the screening since the same flavon fields contribute to both mass matrices. The proportionality of Eq. (44) is rather a generic requirement for several schemes, so we discuss it in some details. This proportionality can be obtained in various ways as follows:

(i) All the coupling constants for 10_H and independently $\overline{16}_H$ are equal to each other (or approximately equal):

$$y_{10} = y'_{10} = y''_{10}, \quad y_{16} = y'_{16} = y''_{16}.$$

In this case correlations are not needed. The equalities can be residuals of embedding of A_4 into $SO(3)$, so that the singlet of A_4 ($1, 1', 1''$) steam from the triplet representation of $SO(3)$. The (approximate) equality of the coupling constants is also needed to obtain the hierarchy of masses of quarks and charged leptons.

(ii) The proportionality can be a consequence of further unification of 16_F and S in 27-plets of E_6 .

In both cases (i) and (ii) the Z_4 charges of 16_F and S should be equal, as given in Eq. (43).

(iii) The proportionality can be a consequence of symmetry with respect to permutation $16_F \leftrightarrow S$ and $10_H \leftrightarrow \overline{16}_H$. In this case also Z_4 charges of 10_H and $\overline{16}_H$ should be the same, which is satisfied in the symmetry assignment.

Under condition (44) the matrices m_D and M_D have the same flavor structure, so that $m_D/M_D \propto \mathbb{1}$.

Due to the absence of singlet flavons, M_S should have all equal diagonal elements from the bare mass terms. The VEV alignment of the triplet $\langle \xi \rangle = u(1, 1, 1)$ leads to the matrix M_S in the form

$$M_S = M_{33}^{special} + \Delta M_S,$$

where the special matrix $M_{33}^{special}$ is given in (25) with $\mu = M_S^0$ and $\beta = h_1 u$. A perturbation matrix ΔM_S is needed, and should be of the form ΔM in Eq. (27), so that the total M_S has the TBM form and thus is diagonalized by U_{TBM} . The eigenvalues of M_S equal

$$\lambda_{1,3} = M_S^0 - h_1 u \pm M_0, \quad \lambda_2 = M_S^0 + 2h_1 u + M_0,$$

and there is enough freedom to get any spectrum⁴. The light neutrino masses satisfy that $m_i \propto \lambda_i$.

There are several ways to obtain the additional correction ΔM_S in the form of ΔM :

1). A correction proportional to ΔM but directly contributing to m_ν can come from additional fermion singlets S, S', S'' , as we will discuss in sect. 5.2.

2). Another modification of the scheme which would be equivalent to the correction ΔM (27) is to introduce flavons ϕ, ϕ', ϕ'' which modify the diagonal elements of M_S . Suppose that ϕ, ϕ', ϕ'' acquire VEVs (v_1, v'_1, v''_1) which satisfy together with couplings the equality $y_1 v_1 = y'_1 v'_1 = y''_1 v''_1$. And differently from above, $\langle \xi \rangle = (u_1, u_2, u_3)$. Then the mass matrix of singlets can have the form

$$M_S = h_1 \begin{pmatrix} 3y_1 v_1/h_1 & u_3 & u_2 \\ \dots & 0 & u_1 \\ \dots & \dots & 0 \end{pmatrix} + M_S^0 \mathbb{1}. \quad (45)$$

In the case of complete screening it gives the light neutrino mass matrix: $m_\nu = b \cdot M_S$. Imposing further equality $u_3 \equiv u_2$ we obtain from (45) maximal 2-3 mixing, and still enough freedom is left to fit all the data. In particular, equality $h_1 u_1 = h_1 u_2 + 3y_1 v_1$ leads to the TBM mixing and to the mass eigenvalues

$$m_1 = b(M_S^0 + h_1 u_1 - 2h_1 u_2), \quad m_2 = b(M_S^0 + h_1 u_1 + h_1 u_2), \quad m_3 = b(M_S^0 - h_1 u_1),$$

which can reproduce the required mass spectrum.

⁴Notice that $M_0 \neq 0$ is important, otherwise we would have two degenerate values $\lambda_{1,3} = M_S^0 - h_1 u$. That would lead to $m_1 = m_3$ for light neutrinos, which contradicts observations.

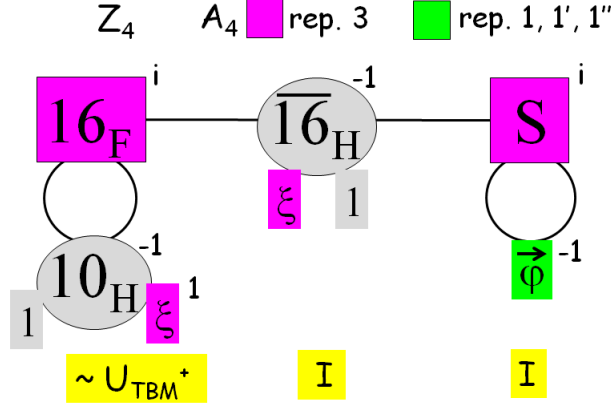


Figure 2: The same as in Fig. 1, but for the triplet - triplet flavon scheme. Grey box with 1 corresponds to flavonless couplings.

3.2 Triplet - triplet flavon scheme

The Lagrangian is given in Eq. (42) with $\vec{y}_{10} = \vec{y}_{16} = 0$ and graphic representation of the scheme is shown in Fig. 2. It allows to realize a scenario

$$U_l \approx U_{TBM}^\dagger, \quad m_D/M_D \propto \mathbb{1}, \quad U_S = \mathbb{1},$$

in which the mixing originates from the charged leptons. Vanishing singlet couplings \vec{y}_{10} and \vec{y}_{16} can be obtained by imposing the Z_4 assignment

$$\{16_F, S|10_H, \overline{16}_H|\vec{\varphi}, \xi\} \sim \{i, i|-1, -1|-1, 1\}. \quad (46)$$

Now only the triplet and operators without flavons contribute to the visible and portal masses. Therefore

$$m_D = m_D^0 \cdot \mathbb{1} + r M_{33}^\xi, \quad M_D = M_D^0 \cdot \mathbb{1} + M_{33}^\xi, \quad (47)$$

where $m_D^0 = y_{10}^b \langle 10_H \rangle$, $m_D^0 = y_{16}^b \langle \overline{16}_H \rangle$ and $r \equiv h_{10} \langle 10_H \rangle / h_{16} \langle \overline{16}_H \rangle$.

Diagonalizing m_D we find that masses of charged leptons can be reproduced if

$$|m_D^0| = \frac{1}{3}(m_e + m_\mu + m_\tau), \quad (h_{10})^2 \sum_i u_i^2 \approx \frac{1}{3} m_\tau^2, \quad (h_{10})^3 u_1 u_2 u_3 \approx \frac{1}{27} m_\tau^3. \quad (48)$$

The second and third conditions are satisfied if, e.g. $u_1 \approx u_2 \approx u_3 \approx m_\tau / (3h_{10})$.

One important feature of this case is that the matrices m_D and M_D in Eq. (47) have different eigenvalues but the same mixing. Since off-diagonal parts of both matrices are generated by the same flavon field ξ , they are diagonalized by the same rotation U_{33} which diagonalizes M_{33}^ξ : $U_{33}^T M_{33}^\xi U_{33} = M_D^{diag}$ (see Appendix. C). At the same time, depending on relative size of diagonal and off-diagonal contributions, the matrices m_D and M_D can have different hierarchies of eigenvalues. Consequently, $U_l = U_{33}$ and in the flavor basis the mass matrix of light neutrinos becomes

$$m_\nu = m_D^{diag} (M_D^{diag})^{-1} U_{33}^T M_S U_{33} (M_D^{diag})^{-1} m_D^{diag}. \quad (49)$$

Complete screening, $m_D(M_D)^{-1} = c \cdot \mathbb{1}$, requires that $y_{10}^b : y_{16}^b = h_{10} : h_{16}$. In this case we obtain from Eq. (49)

$$m_\nu = c^2 U_{33}^T M_S U_{33} = c^2 U_l^T M_S U_l \quad (50)$$

in the flavor basis.

In the hidden sector due to Z_4 symmetry given by Eq. (46) $h_1 = 0$, and so the flavon triplet does not contribute to the singlet mass matrix. Therefore $M_S = M_{33}^\varphi$ and the neutrino matrix in Eq. (50) is diagonalized by U_{33}^\dagger . Consequently, the neutrino mixing $U_0 = U_{33}^\dagger$. This result can be immediately obtained by noticing that in the original symmetry basis and in the presence of complete screening the neutrino mass matrix is diagonal whereas the lepton mass matrix is diagonalized by $U_l = U_{33}$.

In turn, the matrix U_{33} can reproduce the TBM or BM mixing. Indeed, if the 1-3 mixing is generated by the CKM physics, the 1-3 mixing, which originates from U_{33} , should be zero. So, in terms of the standard parametrization of U_{PMNS} we have to have $U_{33} = U_{12}(\theta_{12})^T U_{23}(\theta_{23})^T$. This requires equality $u_2 = u_1$ in M_{33}^ξ (24) which leads to the maximal 1-2 mixing, $\theta_{12} = \pi/4$. Then for the 2-3 mixing we obtain

$$\tan 2\theta_{23} = \frac{2\sqrt{2}u_1}{u_3}.$$

For $u_1 \gg u_3$ this equation would give nearly maximal 2-3 mixing. However, this is not consistent with (48) which has for $u_2 = u_1$ two solutions: $u_3 = u_1$ and $u_3 = (2\sqrt{2}i)u_1$. In the first (best) case we obtain $\theta_{23} = 35^\circ$, which can be slightly bigger if $u_2 \neq u_1$, but the latter corresponds to non-zero 1-3 mixing. Thus, the scheme produces the bi-large mixing from the neutrino sector. Neutrino masses are given by eigenvalues of M_{33}^{diag} .

3.3 Singlet - triplet flavon scheme

The Lagrangian of this scheme is given by Eq. (42) with $h_{10} = \vec{y}_{16} = 0$, and the graphic representation is shown in Fig. 3. Only flavon singlets contribute to the Dirac mass matrix and only triplet flavons to the portal mass matrix. Here we can realize the scenario

$$U_l = \mathbb{1}, \quad M_D \propto M_S = M_{33}^\xi$$

with cancellation of the portal and hidden sector matrices in the double seesaw mechanism. Vanishing h_{10} and \vec{y}_{16} can be obtained with Z_4 symmetry assignment

$$\{16_F, S|10_H, \overline{16}_H|\vec{\varphi}, \xi\} \sim \{i, -i|-1, -1|1, -1\}. \quad (51)$$

The key feature is that 16_F and S have different Z_4 charges. The Z_4 symmetry also forbids the low-dimension (flavonless) contribution of the mass M_D^0 , so that

$$m_D = m_D^0 \mathbb{1} + M_{33}^\varphi, \quad M_D = M_{33}^\xi,$$

where $m_D^0 = y^b \langle 10_H \rangle$. The matrix m_D has enough free parameters to reproduce the charged lepton masses.

With the assignment in Fig. 3, only triplet ξ contributes to the hidden sector matrix, so that $M_S \propto M_{33}^\xi$. Furthermore, since the same ξ contributes to both M_S and M_D we have

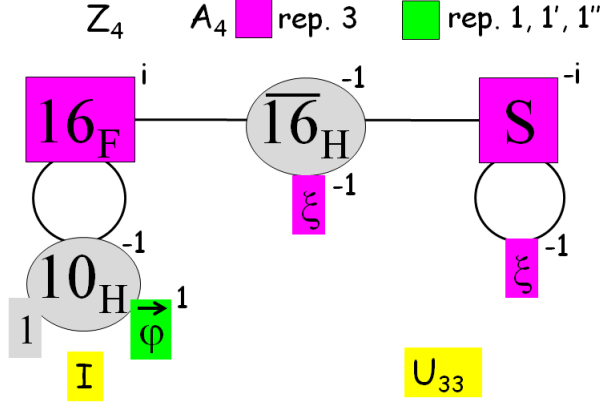


Figure 3: The same as in Fig. 1, but for the singlet - triplet flavon scheme. The matrix which diagonalize m_ν comes from a non-trivial combination of U_{33} and m_D^{diag} (see the text).

$M_S \propto M_D$. Therefore M_S cancels with one inverse M_D in the double seesaw formula. As a result, the light neutrino mass matrix becomes

$$m_\nu \propto m_l^{diag} (M_{33}^\xi)^{-1} m_l^{diag}. \quad (52)$$

Effectively the expression for mass looks like a single seesaw and only the overall scale of the RH neutrino masses imprints information about the presence of the hidden sector.

In spite of strong hierarchy in m_l^{diag} the neutrino mass matrix in (52) can be made in agreement with data due to the special form of M_{33}^ξ , such that its inverse matrix,

$$(M_{33}^\xi)^{-1} = \frac{1}{2hu_1u_2u_3} \begin{pmatrix} -u_1^2 & u_1u_2 & u_1u_3 \\ \dots & -u_2^2 & u_2u_3 \\ \dots & \dots & -u_3^2 \end{pmatrix}, \quad (53)$$

has quadratic hierarchy in u_i . To show this we take $m_l^{diag} = m_\tau \text{diag}(\lambda^6, \lambda^2, 1)$, where $\lambda \equiv \sin \theta_C \simeq 0.22$ is the Cabibbo angle. Then for VEV the hierarchy $\langle \xi \rangle = (u_1, u_2, u_3) = (-1, \lambda^3, \lambda^5)$ with coefficients of the order 1 (which is even weaker than the hierarchy in m_l^{diag}) Eqs. (52) and (53) lead to

$$m_\nu \propto \begin{pmatrix} -\lambda^4 & \lambda^3 & \lambda^3 \\ \dots & \lambda^2 & -\lambda^2 \\ \dots & \dots & \lambda^2 \end{pmatrix}.$$

This matrix produces a zero (small) θ_{13} , maximal 2-3 mixing and large 1-2 mixing. It only corresponds to the normal mass hierarchy.

3.4 Triplet - singlet flavon scheme

This case is in some sense opposite to the previous one: the triplet ξ produces the Dirac mass matrix, whereas singlets generate the portal mass matrix, see Fig. 4. The Lagrangian

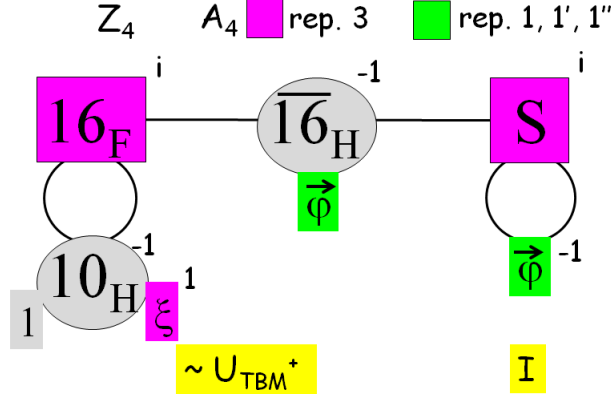


Figure 4: The same as in Fig. 1, but for the triplet - singlet flavon scheme.

is given by Eq. (42) with $\vec{y}_{10} = h_{16} = 0$. Vanishing \vec{y}_{10} and h_{16} can be obtained by the Z_4 with assignment

$$\{16_F, S | 10_H, \overline{16}_H | \vec{\varphi}, \xi\} \sim \{i, -i | -1, -1 | -1, 1\}.$$

It leads to

$$m_D = m_D^0 \cdot \mathbb{1} + M_{33}^\xi, \quad M_D = M_{33}^\varphi.$$

Symmetry allows flavonless contribution m_D^0 , whereas a similar contribution to M_D is forbidden. Generation of the Dirac mass matrix m_D is similar to the case in sect. 3.2.

The scheme allows to realize an interesting possibility when mixing comes essentially from the RH rotations which diagonalize the neutrino Dirac mass matrix. This is possible due to the Majorana nature of neutrinos. If $M_D \propto \mathbb{1}$ (only the true singlet, φ , contributes), then

$$m_\nu \propto m_D^{diag} \cdot U_{33}^\dagger M_S U_{33}^* \cdot m_D^{diag}$$

in the flavor basis, and $m_D^{diag} = m_l^{diag}$.

Due to Z_4 only $\vec{\varphi}$ can couple to S , and consequently, M_S is diagonal. Even in this case by selecting parameters of U_{33} one can reproduce observables.

3.5 Scheme with mixed contributions: the TBM mixing

Here and in the next subsection we consider general situations when both triplet and singlet flavons interact in the visible sector. The model in this subsection realizes the scenario in which

$$U_l = U_\omega^*, \quad m_D^T M_D^{-1} \propto \mathbb{1}, \quad U_S = U_{13}^{max}.$$

So, U_ω appears from the charged leptons mass matrix whereas the hidden sector gives maximal 1-3 mixing, and there is a complete screening. As a result, $U_0 \propto U_\omega U_{13}^{max} = U_{TBM}$.

The symmetry assignments of the field content we use is

$$\{16_F, S | 10_H, \overline{16}_H | \vec{\varphi}, \phi, \xi, \xi'\} \sim \{1, i | i, 1 | -i, -1, -i, -1\}$$

under Z_4 , and the full graphic representation are shown in Fig. 5.

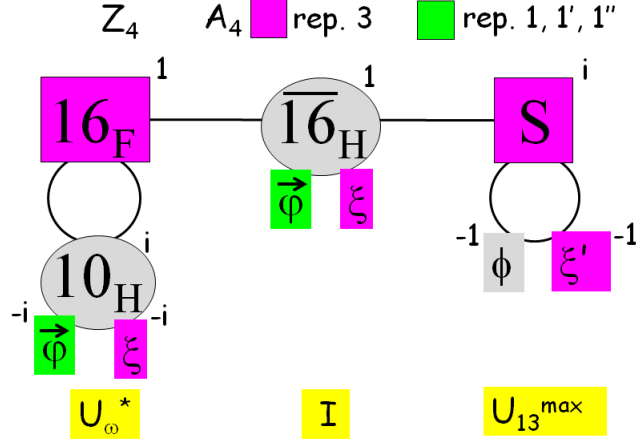


Figure 5: The same as in Fig. 1 but for the mixed flavon scheme for the TBM mixing.

Both the singlet $\vec{\varphi}$ and triplet ξ flavons, with arbitrary VEVs, participate in visible and portal interactions. The others, ϕ and ξ' , exclusively couple with fermion singlets.

The Lagrangian has the following terms

$$\begin{aligned}
\mathcal{L}_D &= \frac{10_H}{\Lambda_f} [\vec{y}_{10} (16_F \cdot 16_F)_\varphi \vec{\varphi} + h_{10} (16_F \cdot 16_F \cdot \xi)], \\
\mathcal{L}_{NS} &= \frac{\overline{16}_H}{\Lambda_f} [\vec{y}_{16} (16_F \cdot S)_\varphi \vec{\varphi} + h_{16} (16_F \cdot S \cdot \xi)], \\
\mathcal{L}_S &= y_1 (S \cdot S) \phi + h'_1 (S \cdot S \cdot \xi').
\end{aligned} \tag{54}$$

The Z_4 symmetry forbids all the interactions without flavons including the bare mass terms.

For the Dirac mass matrix we obtain

$$m_D = \frac{\langle 10_H \rangle}{\Lambda_f} \begin{pmatrix} M_1 & h_{10} u_3 & h_{10} u_2 \\ \dots & M_2 & h_{10} u_1 \\ \dots & \dots & M_3 \end{pmatrix},$$

where M_i are defined in Eq. (21). To be diagonalized by U_ω this matrix should be of the form Eq. (34). This requires that $M_i = h_{10} u_i$, or according to Eq. (21), the relation (39) with $h = h_{10}$ and $\vec{y} = \vec{y}_{10}$.

Using this VEV relation, the mass matrix can be written as

$$m_D = \frac{h_{10} \langle 10_H \rangle}{\Lambda_f} \begin{pmatrix} u_1 & u_3 & u_2 \\ \dots & u_2 & u_1 \\ \dots & \dots & u_3 \end{pmatrix}.$$

It is diagonalized as $U_l^\dagger m_D V_R$, where $U_l = U_\omega^*$ and $V_R = U_\omega$. The eigenvalues of the matrix equal

$$(m_{D1}, m_{D2}, m_{D3})^T = \frac{h_{10} \langle 10_H \rangle}{\Lambda_f} (\sqrt{3} U_\omega^*) (u_1, u_2, u_3)^T = \frac{3 \langle 10_H \rangle}{\Lambda_f} (y_{10} v, y'_{10} v', y''_{10} v'')^T,$$

so the mass hierarchy requires that $y_{10}v \ll y'_{10}v' \ll y''_{10}v''$.

The same flavons $\vec{\varphi}$ and ξ appear in \mathcal{L}_D and \mathcal{L}_{NS} . Therefore, the flavor structures of mass matrices m_D and M_D are the same provided that the Yukawa couplings of 10_H and 16_H are proportional to each other

$$y_{10} : y'_{10} : y''_{10} : h_{10} = y_{16} : y'_{16} : y''_{16} : h_{16}. \quad (55)$$

That is, now couplings of $\vec{\varphi}$ and ξ should also correlate. Possible origins of this proportionality have been discussed in sect. 3.1. As a consequence,

$$m_D^\nu = m_l = m_D = \frac{y_{10}\langle 10_H \rangle}{y_{16}\langle \overline{16}_H \rangle} M_D, \quad (56)$$

and therefore $m_D M_D^{-1T} = y_{10}\langle 10_H \rangle / (y_{16}\langle \overline{16}_H \rangle) \cdot \mathbb{1}$, reproducing the complete screening.

Maximal 1-3 mixing should be obtained from M_S in the form of M_{33} . If $\langle \phi \rangle = v_1$ and the VEV of the triplet is aligned as $\langle \xi' \rangle \rightarrow (0, u', 0)$, we obtain

$$M_S = \begin{pmatrix} y_1 v_1 & 0 & h'_1 u' \\ \dots & y_1 v_1 & 0 \\ \dots & \dots & y_1 v_1 \end{pmatrix}.$$

The eigenvalues of M_S equal $y_1 v_1 \pm h'_1 u'$ and $y_1 v_1$, and consequently, the light neutrino masses equal

$$m_{1,3} = B(y_1 v_1 \pm h'_1 u'), \quad m_2 = B y_1 v_1, \quad B \equiv \frac{y_{10}^2 \langle 10_H \rangle^2}{y_{16}^2 \langle \overline{16}_H \rangle^2}. \quad (57)$$

This gives normal mass ordering with $m_1 : m_2 : m_3 \simeq 1 : 1 : 3$, if $h'_1 u' \sim -2y_1 v_1$. The inverted mass ordering can be obtained if two additional flavon singlets, ϕ' and ϕ'' , are introduced in the hidden sector with couplings satisfying $y'_1 \langle \phi' \rangle = \omega y''_1 \langle \phi'' \rangle$. This adds one free parameter to M_S , and thus, allows arbitrary neutrino masses without changing maximal 1-3 mixing.

The model can be simplified if certain high-order corrections to m_D are added. Assume that $\langle \varphi' \rangle = \langle \varphi'' \rangle = 0$, or simply that φ' and φ'' are removed from the scheme. In practice, this is equivalent to the case $\vec{y}_{10} = 0$ but $m_D^0 \neq 0$. If then

$$\langle \xi \rangle = u(1, 1, 1),$$

we obtain

$$m_D = M_{33}^{special}$$

of Eq. (25) with $\mu = y_{10}v$ and $\beta = h_{10}u$. The equality (56) and M_S remain unchanged.

To break the degeneracy of eigenvalues of m_D and fix $U_l = U_\omega$, one has to introduce a non-symmetric correction δm_D (see e.g. $\Delta M'$ in Eq. (27)). In particular, one can use the anti-symmetric correction

$$\delta m_D = \delta m \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

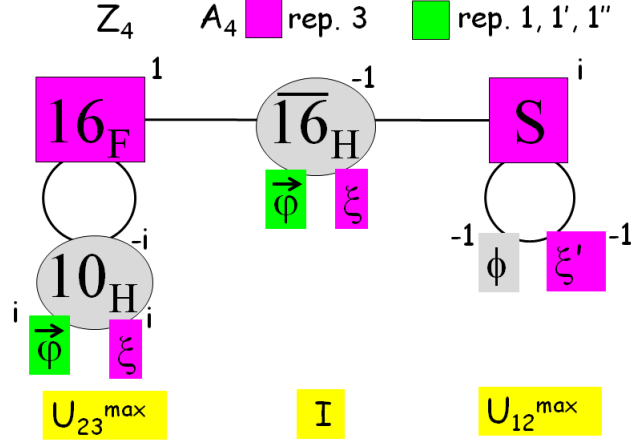


Figure 6: The same as in Fig. 1 but for the mixed flavon scheme for the BM mixing.

which originates from the effective 120_H . The latter appears from non-renormalizable interactions

$$\frac{1}{\Lambda} 16_F 16_F 16_H 16_H, \quad 16_F 16_F 45_H 10_H,$$

noticing that the decomposition of $(16_H \times 16_H)$ contains 120_H , see [29, 69, 70] for examples. As a result, $U_{PMNS} = U_\omega U_{13}^{\max}$ with non-degenerate masses of charged leptons given by

$$m_{D1} = y_{10}v + 2h_{10}u, \quad m_{D2,D3} = y_{10}v - h_{10}u \mp i\sqrt{3}\delta m.$$

To keep screening, similar corrections should also be introduced for M_D . It is straightforward to see that the masses of light neutrinos are the same as in Eq. (57), because M_S is unchanged.

3.6 Scheme with mixed contributions: the BM mixing

Our framework also allows for new possibilities to produce the BM mixing (see Appendix. D). The BM mixing can be factorized by two maximal mixing rotations: $U_{BM} = U_{23}^{\max} U_{12}^{\max}$. In this connection we present a scenario in which maximal 2-3 rotation comes from the charged leptons and the maximal 1-2 one arises from the hidden sector:

$$U_l = (U_{23}^{\max})^T, \quad m_D(M_D)^{-1} \propto \mathbb{1}, \quad U_S = U_{12}^{\max}.$$

The particle content and the charge assignments are given in Fig. 6, where the Z_4 charges are

$$\{16_F, S | 10_H, \overline{16}_H | \vec{\varphi}, \phi, \xi, \xi'\} \sim \{1, i | -i, -1 | i, -1, i, -1\},$$

respectively. All the mass matrices are of the type M_{33} . Bare mass terms and operators without flavons are forbidden by Z_4 symmetry. The symmetry assignment allows flavon singlets $\vec{\varphi}$ and triplet ξ to couple with fermions in visible and portal interactions, while the other two flavons, singlet ϕ and triplet ξ' , appear in the hidden sector only.

To get maximal 2-3 mixing from the charged leptons, where $m_l = m_D$, we impose equalities

$$y'_{10} = y''_{10}, \quad v' = v'', \quad \langle \xi \rangle = (u, 0, 0). \quad (58)$$

These equalities imply the permutation symmetry ($' \leftrightarrow ''$) in the visible and portal sectors, and therefore lead to maximal 2-3 mixing. Under conditions (58) we obtain

$$m_D = \frac{\langle 10_H \rangle}{\Lambda_f} \begin{pmatrix} y_{10}v + 2y'_{10}v' & 0 & 0 \\ 0 & y_{10}v - y'_{10}v' & h_{10}u \\ 0 & h_{10}u & y_{10}v - y'_{10}v' \end{pmatrix}.$$

The eigenvalues of this matrix equal

$$m_{D1} = \frac{\langle 10_H \rangle}{\Lambda_f} (y_{10}v + 2y'_{10}v'), \quad m_{D2, D3} = \frac{\langle 10_H \rangle}{\Lambda_f} (y_{10}v - y'_{10}v' \mp h_{10}u). \quad (59)$$

To obtain mass hierarchy we should arrange that $y'_{10}v' \approx -0.5y_{10}v$, which leads to

$$m_{D2} \simeq \frac{\langle 10_H \rangle}{\Lambda_f} \left(\frac{3}{2}y_{10}v - h_{10}u \right), \quad m_{D3} \simeq \frac{\langle 10_H \rangle}{\Lambda_f} \left(\frac{3}{2}y_{10}v + h_{10}u \right),$$

and $m_{D1} \ll m_{D2}$. Additional relation $h_{10}u \approx \frac{3}{2}y_{10}v$ gives $m_{D2} \ll m_{D3}$.

To realize complete screening, $m_D \propto M_D$, we impose the proportionality of both singlet and triplet couplings \vec{y} and h with 10_H -plets and $\overline{16}_H$ -plets as Eq. (55).

The maximal 1-2 mixing from M_S requires that the second flavon triplet, interacting in the hidden sector, has VEV $\langle \xi' \rangle \rightarrow (0, 0, u')$, which implies the permutation symmetry of the first and second generations in hidden sector. Equal diagonal elements of M_S are generated by singlet ϕ with VEV $\langle \phi \rangle = v_1$. Under these conditions

$$M_S = \begin{pmatrix} y_1v_1 & h'_1u' & 0 \\ h'_1u' & y_1v_1 & 0 \\ 0 & 0 & y_1v_1 \end{pmatrix}.$$

Since $m_\nu \propto M_S$, an agreement with observed mass spectrum can be achieved for the inverted mass hierarchy when $|y_1v_1| \ll |h'_1u'|$. As a result, the three light neutrino masses equal $m_3 \propto y_1v_1$, $m_2 \approx m_1 \propto h'u'$ and $\Delta m_{21}^2 \propto 4y_1v_1h'_1u'$.

To obtain normal mass hierarchy we can introduce also ϕ' and ϕ'' with couplings $y'_1\langle\phi'\rangle = \omega y''_1\langle\phi''\rangle$, which actually allows to obtain arbitrary values of neutrino masses. Recall that conditions we impose here can be consequences of additional symmetries operating in the hidden sector.

4 Schemes with S transforming as singlets of A_4

For $\vec{S} = (S, S', S'') \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ the portal and the hidden parts of the Lagrangian change with respect to Eq. (42):

$$\begin{aligned} \mathcal{L}_{NS} &= \frac{\overline{16}_H}{\Lambda_f} \vec{h}_{16} (16_F \cdot \xi)_S \vec{S}, \\ \mathcal{L}_S &= M_S^0 SS + 2\tilde{M}_S^0 S'S'' + (y_{11} SS + 2y_{23} S'S'') \varphi \\ &+ (y_{22} S'S' + 2y_{13} SS'') \varphi' + (2y_{12} SS' + y_{33} S''S'') \varphi''. \end{aligned} \quad (60)$$

The visible interactions are the same as before, so that the Dirac mass matrices have two contributions: one from singlets $\bar{\varphi}$ and the other from triplet ξ of A_4 . Since the fermion singlets S transform as the three one-dimensional representations of A_4 , the portal term has only one contribution: from triplet ξ . Therefore, in contrast to the schemes in the previous section with $S \sim \mathbf{3}$, now we can only have two possibilities for zero couplings related to the Dirac terms: $h_{10} = 0$ or $\vec{y}_{10} = 0$. For both possibilities, according to Eqs. (30-32) the portal and hidden sector mass matrices are given by (in the case of only one triplet flavon)

$$M_D = M_{31} = \sqrt{3} D_u U_\omega D_h, \quad M_S = M_{11}. \quad (61)$$

Since 16_F and S are in different representations of A_4 , the corresponding mass matrices have different flavor structures and it becomes non-trivial to get certain correlations between them.

4.1 Scheme with $h_{10} = 0$

Vanishing h_{10} can be achieved, e.g. due to Z_2 symmetry, under which only ξ and $\overline{16}_H$ are odd. Only singlet flavons contribute to the visible sector masses and therefore $m_D = m_D^0 \mathbb{1} + M_{33}^\varphi$ with elements fixed by masses of charged leptons (or up-type quarks), *i.e.* $m_D = m_l = m_l^{diag}$. So, mixing is generated by neutrinos. The light neutrino mass matrix can be written as

$$m_\nu = m_l^{diag} (M_{31}^T)^{-1} M_S (M_{31})^{-1} m_l^{diag},$$

or explicitly, according to Eq. (61)

$$m_\nu = \frac{1}{3} \frac{m_l^{diag}}{D_u} U_\omega^* \frac{1}{D_h} M_S \frac{1}{D_h} U_\omega \frac{m_l^{diag}}{D_u}. \quad (62)$$

For $D_h \propto \mathbb{1}$ and $m_l^{diag} (D_u)^{-1} \propto \mathbb{1}$ (partial screening), the expression (62) is reduced to

$$m_\nu \propto \frac{1}{3} U_\omega^* M_S U_\omega.$$

If the bare mass term dominates in M_S we would get

$$m_\nu = \frac{1}{9} \left[3 \tilde{M}_S^0 \mathbb{1} + \left(M_S^0 - \tilde{M}_S^0 \right) M_{dem} \right],$$

where M_{dem} is the matrix with all the elements equal 1. It can be diagonalized by the TBM mixing but has two equal neutrino masses. Such a degeneracy means that the mixing is not fixed completely. More contributions are required to break it. A straightforward possibility is to use additional contributions to M_{11} in order to obtain maximal 1-3 mixing from M_S , so that $U_0 = U_\omega U_{13}^{max} = U_{TBM}$.

In what follows we will consider another way of getting the required corrections, which also justifies some assumptions we have made before. We elaborate on a scenario which realizes relations

$$m_D = diag, \quad m_D^T M_D^{-1} \propto U_\omega^*, \quad D_h^{-1} M_S D_h^{-1} = M_{13}^{max}.$$

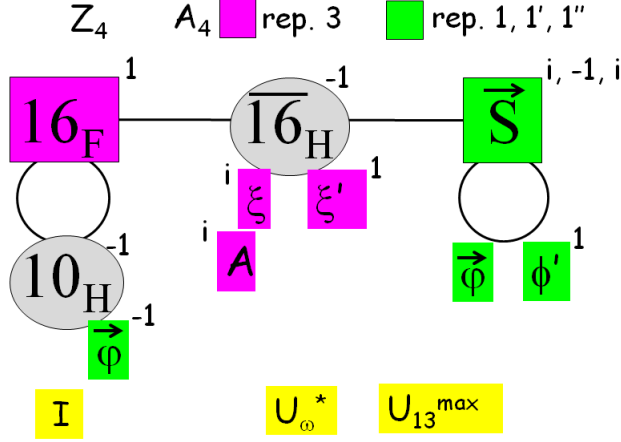


Figure 7: The same as in Fig. 1 but for the flavon scheme with S in three singlet representations of A_4 and $h_{10} = 0$.

Here, as in the previous case, the first relation means that $U_l = \mathbb{1}$, the second relation corresponds to partial screening of the Dirac structures which generates the magic matrix. The singlet matrix together with D_h produces maximal 1-3 mixing. Then $U_0 \propto U_\omega U_{13}^{max} = U_{TBM}$.

We introduce Z_4 to forbid h_{10} and some other couplings. The field content, symmetry assignments and graphic representation of the scheme are given in Fig. 7. The features of the scheme (as compared with minimal structure discussed in the beginning of the subsection) are (i) different Z_4 charges of fermion singlets, (ii) two flavon triplets participating in the portal interactions, (iii) additional flavon singlet ϕ' , (iv) auxiliary flavon triplet A .

The most general expressions for the three terms of the Lagrangian (2) allowed by symmetry in the lowest order are

$$\begin{aligned}
\mathcal{L}_D &= \frac{10_H}{\Lambda_f} \vec{y}_{10} (16_F \cdot 16_F)_\varphi \vec{\varphi}, \\
\mathcal{L}_{NS} &= \frac{\overline{16}_H}{\Lambda_f} [h_{16} (16_F \cdot \xi) S + h'_{16} (16_F \cdot \xi')'' S' + h''_{16} (16_F \cdot \xi)' S''], \\
\mathcal{L}_S &= y_1 (S \cdot S) \varphi + 2y'_1 (S \cdot S'') \varphi' + y''_1 (S'' \cdot S'') \varphi'' + \tilde{y}'_1 (S' \cdot S') \phi'. \quad (63)
\end{aligned}$$

Notice that Z_4 forbids the low-dimension flavonless term $16_F 16_F 10_H$ since 10_H has the charge -1 . It also ensures that triplet flavons do not couple with $16_F 16_F$, and therefore only singlets contribute to the Dirac mass matrix, leading to $m_D = M_{33}^\varphi$. The symmetry Z_4 also forbids the bare mass terms (SS) and $(S'S'')$.

There is a generic problem here: On the one hand, obtaining U_ω from M_D requires (in the simplest case) that only one flavon triplet participates in the portal interactions. On the other hand, to get maximal 1-3 mixing from the hidden sector one needs to distinguish S' from S and S'' , so that the terms $(SS')\varphi''$ and $(S'S'')\varphi$ are forbidden. To forbid such terms, singlets S and S'' should have equal Z_4 charges which differ from the charge of S' . Therefore in \mathcal{L}_{NS} two flavon triplets ξ and ξ' carrying different Z_4 charges are necessary. Moreover, a

new flavon singlet, ϕ' , with Z_4 charge different from $\vec{\varphi}$, should be introduced to generate the mass of S' .

Now in the visible sector, both the bare mass and off-diagonal elements of M_{33} are absent, so \mathcal{L}_D in Eq. (63) only generates the same diagonal Dirac mass matrices $m_D = M_{33}^\varphi(\vec{y}_{10}, \vec{v})$ to all SM fermion components as in Eq. (21)

$$(m_{D1}, m_{D2}, m_{D3})^T = \frac{\langle 10_H \rangle}{\Lambda_f} \sqrt{3} U_\omega (y_{10} v, y'_{10} v', y''_{10} v'')^T. \quad (64)$$

So the expressions of mass matrices are in the flavor basis from the beginning⁵. The hierarchy of mass eigenvalues m_{D1}, m_{D2}, m_{D3} can be obtained in the same way as Eq. (23).

The portal mass matrix generated by the terms \mathcal{L}_{NS} equals

$$M_D = \frac{\langle \overline{16}_H \rangle}{\Lambda_f} \begin{pmatrix} h_{16} u_1 & h'_{16} u'_1 & h''_{16} u_1 \\ h_{16} u_2 & h'_{16} u'_2 \omega & h''_{16} u_2 \omega^2 \\ h_{16} u_3 & h'_{16} u'_3 \omega^2 & h''_{16} u_3 \omega \end{pmatrix}, \quad (65)$$

where the VEVs of two flavon triplet are denoted as $\langle \xi \rangle = (u_1, u_2, u_3)$, $\langle \xi' \rangle = (u'_1, u'_2, u'_3)$. To get exactly the form of M_{31} , their VEVs need to be aligned in the same way: $\langle \xi \rangle = \langle \xi' \rangle$ (*i.e.* $u_i = u'_i$). With this equality, the mass matrix of Eq. (65) can be written as in Eq. (30): $M_D = \sqrt{3} \langle \overline{16}_H \rangle D_u U_\omega D_h / \Lambda_f$.

The mass matrix of singlet fermions generated by Eq. (63) has the form of M_{11} with zeros due to symmetry assignment:

$$M_S = \begin{pmatrix} y_1 v & 0 & y'_1 v' \\ \dots & \tilde{y}'_1 v_1 & 0 \\ \dots & \dots & y''_1 v'' \end{pmatrix}, \quad (66)$$

where $v_1 \equiv \langle \phi' \rangle$.

Combining M_D and M_S we obtain from Eq. (36) the light neutrino mass matrix

$$m_\nu = \frac{1}{3} \left(\frac{\Lambda_f}{\langle \overline{16}_H \rangle} \right)^2 \frac{m_D^{diag}}{D_u} \cdot U_\omega^* \cdot \begin{pmatrix} \frac{y_1 v}{h_{16}^2} & 0 & \frac{y'_1 v'}{h_{16} h''_{16}} \\ \dots & \frac{\tilde{y}'_1 v_1}{h_{16}^2} & 0 \\ \dots & \dots & \frac{y''_1 v''}{h_{16}^2} \end{pmatrix} \cdot U_\omega \cdot \frac{m_D^{diag}}{D_u}. \quad (67)$$

To get the TBM mixing the following two conditions should be fulfilled:

$$m_D^{diag} \propto D_u, \quad (68)$$

which leads to partial screening, and

$$\frac{y_1}{h_{16}^2} v = \frac{y''_1}{h_{16}^2} v'', \quad (69)$$

⁵The diagonal matrix m_D can also be written in the form of product of matrices as $\frac{\langle 10_H \rangle}{\Lambda_f} U_\omega^* (y_{10} v \mathbb{1} + y'_{10} v' C + y''_{10} v'' C^T) U_\omega$, where C is the cyclic matrix with only non-zero (1,3), (2,1), and (3,2) unit elements.

to get maximal 1-3 mixing from the central matrix of Eq. (67).

The first condition (68) is the correlation of the singlet and triplet VEVs, which can be obtained in the same way as it was done in Eq. (41), by introducing the auxiliary A_4 triplet field A with VEVs $\langle A \rangle = (a, a, a)/\sqrt{3}$. The inequality $m_D \ll M_D$ implies a very small VEV: $a \ll 1$. Therefore the auxiliary field does not change other structures of the model. Provided that $y_{10} = y'_{10} = y''_{10}$, the Dirac masses can be rewritten as

$$(m_{D1}, m_{D2}, m_{D3}) = \frac{\sqrt{3}a y_{10} \langle 10_H \rangle}{\Lambda_f} (u_1, u_2, u_3).$$

The second condition, Eq. (69) can be satisfied if, e.g.

$$h_{16} = \omega h''_{16}, \quad y_1 = \omega y''_1, \quad v = \omega v''.$$

These equalities then produce the masses of light neutrinos as

$$m_{1,3} = C \frac{1}{h_{16}} \left(\frac{y_1}{h_{16}} \pm \frac{y'_1}{h''_{16}} \right) v, \quad m_2 = C \frac{\tilde{y}'_1}{h_{16}^2} v_1, \quad C \equiv \frac{a^2 \langle 10_H \rangle^2 y_{10}^2}{\langle 16_H \rangle^2}.$$

Strong mass hierarchy requires $y_1/h_{16} \approx \pm y'_1/h''_{16}$.

There exist different ways to avoid the introduction of the second triplet and tuning of its VEVs. One possibility is to use single ξ with Z_4 charge i and assign to S' the same charge i as other singlets have. Then to get nearly maximal 1-3 mixing from the hidden sector additional symmetry, e.g. Z_2 , can be introduced and made conserved in the hidden sector only (see also discussion in sect. 2.5). If S' is odd and other particles of the hidden sector are even, then M_S will have the form (66) with zero 12- and 23- elements. This additional symmetry is broken in the visible and portal sectors, which have substantially lower energy scale M_{GUT} . So, one expects that corrections to the Z_2 symmetric results are of the order M_{GUT}/M_{Pl} , which are not important.

Instead of second flavon triplet ξ' , second $16'_H$ with Z_4 charge i can be introduced in the portal interaction

$$\frac{h'_{16}}{\Lambda_f} (16_F \xi)'' 16'_H S'.$$

The VEVs of 16-plets should be equal, $\langle 16'_H \rangle = \langle 16_H \rangle$, to get the matrix factor U_ω . Alternatively, to replace ξ' , one can also introduce a flavon singlet ϕ with Z_4 charge $-i$, which leads to a dimension-6 portal interaction $(16_F \xi)'' 16_H \phi S'$.

Notice that this scheme can be made more logical if three $\overline{16}_H$ (or three ξ) with different Z_4 charges are introduced.

4.2 Scheme with $\vec{y}_{10} = 0$

The visible part of the Lagrangian is as in Eq. (42) with $\vec{y}_{10} = 0$ and $h_{10} \neq 0$. The particle content and symmetry alignment are shown in Fig. 8. The triplet flavon contributes to both visible and portal interactions, but structures of m_D and M_D are different. Vanishing \vec{y}_{10} can be obtained with Z_4 assignment

$$\{16_F, S|10_H, \overline{16}_H|\vec{\varphi}, \xi\} \sim \{i, i| -1, -1| -1, 1\},$$

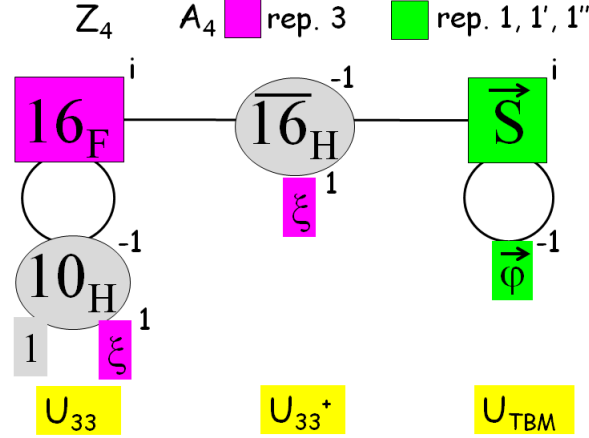


Figure 8: The same as in Fig. 1 but for the flavon scheme with S in three singlet representations of A_4 and $\vec{y}_{10} = 0$.

as in the model of sect. 3.1. Flavor invariant mass m_D^0 is allowed. Consequently, $m_D = m_D^0 \mathbb{1} + M_{33}^\xi$, as in Eq. (47) and $m_l = m_D$. It is thus diagonalized by U_{33} . The portal mass matrix M_D has the form of M_{31} .

The neutrino mass matrix equals

$$m_\nu = \frac{1}{3} m_D \frac{1}{D_u} U_\omega^* \frac{1}{D_h} M_S \frac{1}{D_h} U_\omega^* \frac{1}{D_u} m_D, \quad (70)$$

and it converts to

$$m_\nu = \frac{1}{3} m_D^{diag} \cdot U_{33} \frac{1}{D_u} U_\omega^* \cdot \frac{1}{D_h} M_S \frac{1}{D_h} \cdot U_\omega^* \frac{1}{D_u} U_{33} \cdot m_D^{diag}, \quad (71)$$

in the flavor basis. In the case of two Higgs 10-plets, there is actually $m_D^{diag} = m_u$. In the hidden sector the bare mass terms, M_S^0 and \tilde{M}_S^0 , are forbidden.

If $D_u = \mathbb{1}$, the matrix M_{33} has special form $M_{33}^{special}$, which can be diagonalized by $U_{33} = U_\omega$. In this case Eq. (71) gives

$$m_\nu \propto \frac{1}{3} \frac{m_D^{diag}}{D_h} M_S \frac{m_D^{diag}}{D_h}. \quad (72)$$

Then for partial screening, $m_D^{diag} (D_h)^{-1} \propto \mathbb{1}$, we obtain $m_\nu \propto M_S$. With these conditions we essentially reproduce complete screening. Then M_S should have the TBM form, which can be obtained in the way described by Eqs. (32, 33) in sect. 2.4.

5 Generalizations

We have assumed above that the light neutrino masses are mainly determined by the double seesaw mechanism with three fermion singlets S . Here we consider other possible contributions to masses and mixing, including the CKM physics effects.

5.1 Effects of linear seesaw

The direct mixing between left-handed neutrinos and fermion singlets S via the $\overline{16}_H$ Higgs gives the linear seesaw contribution to the light neutrino masses:

$$m_\nu^{ls} = -m_D \frac{1}{M_D^T} m'^T - m' \frac{1}{M_D} m_D^T,$$

which are the last terms of Eq. (9). Under the condition $M_S \gg M_{\text{cr}}$, where M_{cr} is defined by Eq. (11), this contribution is subleading, but may provide certain required corrections to some observables.

On the contrary, if $M_S \ll M_{\text{cr}}$, the linear seesaw contribution is dominant. As we have mentioned in sect. 2.2 in the presence of only one $\overline{16}_H$, $m' \propto M_D$, and therefore $m_\nu^{ls} \propto m_D$. Thus, no mixing is generated. If more $\overline{16}_H$ multiplets are introduced, the proportionality can be broken: One $\overline{16}_H$ with its VEV along the $SU(2)_L$ singlet direction, $\langle \overline{16}_H \rangle_1$, will be responsible for M_D and another one, $\overline{16}'_H$, with its VEV along the $SU(2)_L$ doublet direction, $\langle \overline{16}_H \rangle_d$, generates the mass m' . If couplings of two 16-plets are different, $y_{16} \neq y'_{16}$, the matrices m' and M_D have different structures, and non-vanishing neutrino mixing can be generated. Notice that for the linear seesaw M_S is irrelevant and m' plays the role of M_S .

As an example, we will briefly describe the case when the linear seesaw gives the main contribution to neutrino mass. We assume that 16_F and S are A_4 triplets. Then $m_l = m_D$ can have the form of M_ω , as Eq. (34), which is diagonalized by U_ω . Also complete screening can be realized, $m_D M_D^{-1T} \propto \mathbb{1}$. For m' we can have

$$m' = M_0 \mathbb{1} + M_{33}^\xi.$$

For certain VEV alignment as in the scheme of sect. 3.5, m' has the form of Eq.(38), producing the maximal 1-3 rotation, so that $U_0 = U_\omega U_{13}^{max} = U_{TBM}$.

Realization of partial screening is different from the case of double seesaw. Let us take a scheme with $16_F \sim \mathbf{3}$ and $\vec{S} \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, in which m_D is diagonal, $m_D M_D^{-1T} \propto U_\omega^*$, and m' has the form of M_{31} . Then the neutrino mass matrix is given by

$$m_\nu^{ls} \propto U_\omega^* m'^T + m' U_\omega^* \propto U_\omega^* D_h U_\omega D_u + D_u U_\omega D_h U_\omega^*.$$

Under conditions $u_3 = u_2$ in D_u and $h_3 = h_2$ in D_h , which imply the 2-3 permutation symmetry, this matrix gives

$$m_\nu^{ls} \propto \begin{pmatrix} 2u_1(h_1 + 2h_2) & (u_1 + u_2)(h_1 - h_2) & (u_1 + u_2)(h_1 - h_2) \\ \dots & 2u_2(h_1 + 2h_2) & 2u_2(h_1 - h_2) \\ \dots & \dots & 2u_2(h_1 + 2h_2) \end{pmatrix},$$

with maximal 2-3 and zero 1-3 mixings. Additional assumption $h_2 = -h_1$ is then required to produce the TBM mixing, since $U_l = \mathbb{1}$. Three parameters h_1 , u_1 and u_2 allow to get arbitrary masses for the three light neutrinos.

5.2 Extended hidden sector

Introduction of more than three fermionic singlets of $SO(10)$ opens up new possibilities to obtain the observed mixing and masses too. And in some cases it allows to simplify the

conditions on couplings and VEVs we imposed in the previous sections. On the other hand, the Dirac screening may not be straightforward any more.

Let us consider for definiteness effects of three additional SO(10) singlets, so that the hidden fermion sector consists of $S \sim (\mathbf{3}, \bar{\mathbf{1}})$ of A_4 . The light neutrino mass matrix is given by

$$m_\nu = (m_D)_{3 \times 3} \cdot \frac{1}{(M_D)_{3 \times n} \cdot (m_S^{-1})_{n \times n} \cdot (M_D^T)_{n \times 3}} \cdot (m_D^T)_{3 \times 3}, \quad (73)$$

where $n = 6$. Taking into account the A_4 symmetry assignment we can write different mass matrices involved as

$$m_D = M_{33}^{(D)}, \quad M_D = \begin{pmatrix} M_{33}^{(P)} & M_{31}^{(P)} \end{pmatrix}, \quad M_S = \begin{pmatrix} M_{33}^{(S)} & M_{31}^{(S)} \\ M_{13}^{(S)} & M_{11}^{(S)} \end{pmatrix}. \quad (74)$$

Consider diagonalization of the inverse matrix $[m_\nu]^{-1}$. Recall that if m_ν is diagonalized by the unitary matrix U , then $[m_\nu]^{-1}$ is diagonalized by U^* . Consequently, for a real U as in the case of U_{TBM} , $[m_\nu]^{-1}$ is diagonalized by the same matrix U . The inverted mass matrix can be written as

$$m_\nu^{-1} = m_D^{-1} M_N m_D^{-1}, \quad (75)$$

where M_N is the effective mass matrix of RH neutrinos:

$$M_N = M_D^T M_S^{-1} M_D. \quad (76)$$

Using expressions (74) we find in terms of 3×3 matrices:

$$M_S^{-1} = \begin{pmatrix} [M_{33} - M_{31} M_{11}^{-1} M_{13}]_S^{-1} & [M_{13} - M_{11} M_{31}^{-1} M_{33}]_S^{-1} \\ [M_{31} - M_{33} M_{13}^{-1} M_{11}]_S^{-1} & [M_{11} - M_{13} M_{33}^{-1} M_{31}]_S^{-1} \end{pmatrix}, \quad (77)$$

where the subscript S means that all the matrices in the bracket belong to the hidden sector. Inserting it into Eq. (76) and then into Eq. (75) we obtain

$$\begin{aligned} m_\nu^{-1} &= (M_{33}^{(D)})^{-1} \left\{ M_{33}^{(P)} [M_{33} - M_{31} M_{11}^{-1} M_{13}]_S^{-1} M_{33}^{(P)} \right. \\ &+ M_{33}^{(P)} [M_{13} - M_{11} M_{31}^{-1} M_{33}]_S^{-1} M_{31}^{(P)T} + M_{31}^{(P)} [M_{31} - M_{33} M_{13}^{-1} M_{11}]_S^{-1} M_{33}^{(P)} \\ &+ \left. M_{31}^{(P)} [M_{11} - M_{13} M_{33}^{-1} M_{31}]_S^{-1} M_{31}^{(P)T} \right\} (M_{33}^{(D)})^{-1}. \end{aligned} \quad (78)$$

Imposing various conditions on the matrices involved, one can get specific structures of m_ν^{-1} and lepton mixing.

Let us consider a special case of $M_{13}^{(S)} \equiv (M_{31}^{(S)})^T = 0$, for which, in fact, we can not use general formula (78) with the inverse of $M_{13}^{(S)}$. In this case $M_{33}^{(S)}$ and $M_{11}^{(S)}$ contribute independently to the mass matrix of the RH neutrinos, *i.e.* the denominator of Eq. (73), through their own portal interactions. Plugging matrices (74) into Eq. (73) we obtain

$$m_\nu = M_{33}^{(D)} \frac{1}{M_{33}^{(P)} [M_{33}^{(S)}]^{-1} M_{33}^{(P)} + M_{31}^{(P)} [M_{11}^{(S)}]^{-1} M_{31}^{(P)T}} M_{33}^{(D)}, \quad (79)$$

whose inverse can be written as

$$[m_\nu]^{-1} = [M_{33}^{(D)}]^{-1} M_{33}^{(P)} [M_{33}^{(S)}]^{-1} M_{33}^{(P)} [M_{33}^{(D)}]^{-1} + [M_{33}^{(D)}]^{-1} M_{31}^{(P)} [M_{11}^{(S)}]^{-1} M_{31}^{(P)T} [M_{33}^{(D)}]^{-1}. \quad (80)$$

In the first term the complete screening can be realized (as in the scheme of sect. 3.2) by

$$[M_{33}^{(D)}]^{-1} M_{33}^{(P)} \propto \mathbb{1},$$

if both matrices are generated by the same triplet ξ only. Furthermore, for the flavon triplet, its VEVs $\langle \xi \rangle = u(1, 1, 1)$, the singlet matrix in hidden sector $M_{33}^{(S)} = M_{33}^{special}$. Therefore the first term in Eq. (80) reduces to $[M_{33}^{special}]^{-1}$. It is straightforward to see that the inverse of $M_{33}^{special}$ has the same special form and therefore can be diagonalized by U_{TBM} .

In the second term of Eq. (80) the partial screening can be realized as in the scheme of Eq.(67): $[M_{33}^{(D)}][M_{31}^{(P)}]^{-1} \propto U_\omega^*$ or $[M_{33}^{(D)}]^{-1} M_{31}^{(P)} \propto U_\omega$. Thus, expression (80) reduces to

$$[m_\nu]^{-1} = [M_{33}^{special}]^{-1} + g U_\omega [M_{11}^{(S)}]^{-1} U_\omega, \quad (81)$$

where g is a coefficient, which takes care of the proportionality and normalization of matrices, thus it is irrelevant for this discussion.

Finally, the proportionality $M_{11}^{(S)} \propto \mathbb{1}$ can be imposed by certain symmetries that only apply to hidden sector interactions. In this case the second term in Eq. (81) becomes $g U_\omega \mathbb{1} U_\omega$, which reproduces exactly the correction ΔM as in Eq. (27). This correction removes ambiguity in diagonalization of the first term and fixes the TBM mixing. There are enough parameters left to reproduce the observed neutrino masses.

5.3 On the CKM new physics

So far we have discussed situations that no CKM mixing is generated and all the Dirac masses of SM fermions are the same: $m_u = m_d = m_l$, contradicting observations. Additional physics (new fields, interactions) is therefore needed to explain properties of the quark sector. This additional physics should not destroy the constructions in the neutrino sector. Here we outline how this can be done, and refer to [45] for more details. To be in agreement with observations, three aspects should be addressed.

1). Difference of the up- and down- type fermions mass hierarchies.

There are various ways to obtain different mass hierarchies of the “upper” and “down” fermions (in the SM doublets) and generate the CKM mixing.

(i) A second 10-plet of Higgses, 10_H^d , can be introduced to give masses of the down fermions, whereas the first one, 10_H^u , generates masses of the upper fermions: up-type quarks and neutrinos. Eq. (3) then results in

$$m_u = m_D = Y_{10}^u v_u, \quad m_d = m_l = Y_{10}^d v_d. \quad (82)$$

Therefore, different Yukawa coupling matrices Y_{10}^u and Y_{10}^d produce different mass hierarchies and also CKM mixing.

To keep the neutrino part unchanged we assume that in the symmetry basis the CKM mixing comes from the down fermions. That is, couplings of 10_H^d further violate A_4 , explicitly or spontaneously.

(ii) Another possibility is to keep the single 10_H but introduce the dimension-5 operator

$$\frac{1}{\Lambda}(16_F 16_H 16_H 16_F).$$

This operator also breaks the mass degeneracy between up- and down- type quarks [69].

(iii) Vector-like families of fermions $10_F, \overline{10}_F$ can exist. Their mixing with usual 16_F break the degeneracy between up-type and down-type masses of quarks. The reason is that 10_F only contains down-type quarks and lepton doublets, but no up-type quarks. The interactions which provide this mixing can originate from renormalizable operators $10_F 16_F 16_H$.

2). Difference of masses of down quarks and charged leptons. To breaks the mass degeneracy between quarks and leptons one can use non-renormalizable terms with additional Higgs multiplets. One option is to introduce 45-plet Higgs scalars 45_H and the dimension-5 operator

$$\frac{1}{\Lambda}(16_F \cdot 10_H^d 45_H \cdot 16_F) \quad (83)$$

with 45_H and 10_H^d only [71]. The product of 10_H^d and 45_H has the decomposition $10 \times 45 = 10 + 120 + 320$, which contains 120_H . If 45_H acquires its VEV in the direction of $(B-L)$, where B (L) is the baryon (lepton) number, then the effective VEV of 120_H leads via interactions (83) to difference of masses of the second fermion generation (e.g. m_μ and m_s).

If more than one 45-plet exist, the dimension-6 operators

$$\frac{1}{\Lambda^2}(16_F \cdot 45_H^a 10_H 45_H^b \cdot 16_F)$$

give contributions to the masses of the first generation of fermions [29, 72].

3). Nearly the same CKM-type mixing of down-type quarks and charged leptons despite the different mass hierarchies of them. In our framework the CKM matrix is generated by m_d , and the CKM type correction to lepton mixing which lead to expression (1) are generated by m_l . Since masses of the down-type quarks and charged leptons are different, the mass matrices m_d and m_l should be different, and therefore in general, one expects that diagonalization matrices U_d and U_l that diagonalize them are also different. This may invalidate Eq. (1), which implies that both matrices should be diagonalized by the same rotation $U_l \simeq U_d$ [45].

One solution to this problem was described before in sect. 3.2. The mass matrix can be the sum of a diagonal matrix proportional to the unit matrix and an off-diagonal matrix:

$$m_0 \mathbb{1} + M_{33}^\xi.$$

The mixing is determined by M_{33}^ξ only, whereas the eigenvalues are given by the total matrix, which depends on m_0 and M_{33}^ξ . So, by varying m_0 , we can change the eigenvalues without modifying the mixing.

6 Conclusion

1. We have considered the lepton mixing and neutrino masses in the framework of SO(10) GUTs with the hidden sector composed of fermionic and bosonic singlets of SO(10), and flavor symmetries realized in both visible and hidden sectors. Two sectors communicate via the portal interactions provided by $\overline{16}_H$ of Higgses.

The framework allows to disentangle the CKM new physics and the neutrino new physics. The neutrino new physics explains the smallness of neutrino masses and generates the large lepton mixing. The CKM new physics, being common for quarks and leptons due to GUT, is responsible for the CKM mixing and difference of mass hierarchies of the up-type and down-type quarks, as well as the charged leptons. The framework naturally realizes the relation between the quark and lepton mixings: $U_{PMNS} \sim V_{CKM}^\dagger U_0$, where the structure of matrix U_0 is determined by the flavor symmetry.

In the lowest-order approximation with neutrino physics only: (i) quark and lepton masses are generated, (ii) no quark mixing appears, (iii) large lepton mixing is produced, whose form is dictated by the flavor symmetry, (iv) small neutrino masses are generated via the double seesaw mechanism, or its generalizations are related to more than three fermionic singlets.

The CKM physics gives corrections (or perturbations) to this lowest-order picture and the corrections are related to further violation of flavor symmetry effects and higher-order operators.

2. We have mainly focussed on the neutrino new physics and generation of U_0 in this paper. Using $A_4 \times Z_4$ as the simplest example of flavor group, we have systematically explored the flavor structures (masses, mixing) which can be obtained in this framework, depending on the field content and symmetry assignment. The interplay of GUT SO(10) symmetry and flavor symmetries is crucial in obtaining certain flavor structures.

In this framework three different mass matrices are involved in generation of lepton mixing: m_D - the Dirac mass matrix of SM fermions (charged leptons and neutrinos), M_D - the portal mass matrix, or $m_D M_D^{-1T}$ - the portal factor, and the matrix of fermion singlets M_S . This leads to new possibilities to explain the observed mixing. One of the key elements, which allows to disentangle the quark masses and mixing from neutrino masses and mixing, is the Dirac screening (either complete or partial).

Generically, even without flavor symmetries, the framework explains no or small quark mixing, large lepton mixing and small neutrino masses. It leads to the relation between the quark and lepton mixings of the form (1).

U_0 in Eq. (1) can be factorized as $U_0 = U_1 U_2 \dots$, where U_i are the matrices of maximal two-generation rotation or matrices of special type, such as the magic matrix U_ω . Then different mass matrices can be responsible for different rotations U_i . In this connection we studied properties of matrices m_D , M_D , M_S generated by different SO(10) Higgs multiplets. Depending on A_4 assignments for fermions they can be of 5 types: M_{33}^{diag} , M_{33}^ξ , M_{31} , M_{11} and $M_0 \cdot \mathbb{1}$.

For a complete set of flavons (triplet and singlets) with arbitrary couplings and VEVs, any (symmetric) structure of M_{33} and M_{11} (and consequently, m_D and M_S) and M_{31} can be obtained. As a consequence, the flavor structure of light neutrino mass matrix is arbitrary.

The matrices acquire certain flavor structures if only specific flavons (e.g. only triplet) contribute to their generation. If the same flavons contribute to both m_D and M_D , the Dirac screening can be obtained. This can be achieved by additional symmetry, e.g. Z_4 . This symmetry forbids certain couplings of flavons, ensuring that different flavons participate in different types of interactions. It can then explain that some flavons appear in the hidden sector only. Or on the contrary, a common symmetry assignment ensures that the same flavons contribute to different mass matrices.

It is also possible that certain symmetries exist in the hidden sector only, being broken (spontaneously or explicitly) by low scale interactions in the visible and portal interactions.

To have a complete determination of masses and mixing, in particular, to obtain $U_0 \sim U_{TBM}$, or U_{BM} , further restrictions are needed. We found the following generic conditions, of which all or some should be satisfied in specific schemes:

- equality (or proportionality) of the flavon singlet couplings \vec{y} in the visible sector (*i.e.* with 10_H) and in portal (with $\overline{16}_H$) interactions;
- equality of flavon singlet VEVs;
- correlation between VEVs of flavon singlets and triplet;
- certain VEV alignment of triplet(s) ξ including equality of three VEVs, or non-zero VEV of only one component of the triplet;
- corrections to special matrices which break degeneracy of mass eigenstates and thus fix the mixing.

These relations can be manifestations of further unification, additional symmetries, or selection of certain minimum of the potential from several (degenerate) possibilities. In some cases correlations of the bare mass terms or flavonless contributions are also required.

3. We have presented several schemes with minimal possible number of flavons and couplings in the visible and portal sectors. Interesting realizations are when U_{TBM} , U_{BM} , or certain factors of these matrices follow from different sectors:

- the TBM mixing from the hidden sector M_S (with complete screening);
- approximate TBM mixing from charged leptons;
- approximate TBM mixing from the portal and hidden sectors;
- the magic matrix from charged leptons and maximal 1-3 mixing from the hidden sector, thus reproducing U_{TBM} ;
- the magic matrix from the portal (partial screening) and maximal 1-3 mixing from the hidden sector;
- the 2-3 maximal mixing from charged leptons and the 1-2 maximal mixing from M_S , thus reproducing U_{BM} .

In many cases explanations of masses and mixing decouple from each other. This is because of the special (maximal, magic) form of the mixing.

We also commented on possible effects of the linear seesaw and additional fermion singlets in the hidden sector. They can provide required corrections to the zero-order structures.

We outlined the CKM physics, indicating additional elements which could be introduced to obtain the quark sector. We showed that they can be introduced in a consistent way without destroying the part related to neutrino new physics.

4. This is an initial study – the first step toward the model building, which shows, nevertheless, interesting potential to explain fermion masses and mixing. In complete models one needs to further introduce the scalar potential explicitly and find its minima, to consider effects of high-order operators, and to take into account renormalization group effects, *etc.* Other flavor symmetry groups can be studied. All these issues are beyond the scope of the present paper.

5. The key challenge is the possibility of testing this framework experimentally. In some cases (or with additional assumptions) one can obtain certain predictions for the neutrino mass hierarchy, the type of mass spectrum, value of the Dirac CP phase and values of Majorana phases. Discovery of the proton decay would be a partial confirmation of the GUT framework. The leptogenesis can give some probes of the framework too.

The hidden sector can provide stable particles to act as dark matter candidates. So, detection of dark matter particles may shed some light on the hidden sector, and consequently, the framework studied in this paper. Sterile neutrinos, if exist, can also originate from such a hidden sector.

A A_4 group

In the context of A_4 flavor symmetry, we use decomposition of product of two triplets a, b as follows

$$\begin{aligned} \mathbf{1} &= (ab) = a_1b_1 + a_2b_2 + a_3b_3, \\ \mathbf{1}' &= (ab)' = a_1b_1 + \omega^2a_2b_2 + \omega a_3b_3, \\ \mathbf{1}'' &= (ab)'' = a_1b_1 + \omega a_2b_2 + \omega^2a_3b_3, \\ \mathbf{3} &= a \cdot b = h_1 \cdot (a_2b_3, a_3b_1, a_1b_2) + h_2 \cdot (a_3b_2, a_1b_3, a_2b_1), \end{aligned} \quad (84)$$

where $\omega = e^{2\pi i/3}$, and h_1 and h_2 are both free number parameters. We further assume that $h_1 = h_2$ is protected by some underlying mechanism, and in practice choose the value of h_1 to be 1 because it can be absorbed into relevant Yukawa couplings. Generally, this assumption does not affect the forms of m_D and M_S , which are required to be symmetric anyway. But in the case that both 16_{Fi} and S_i are triplets under A_4 , this assumption leads to a symmetric M_D , and thus helps to preserve the screening condition which links M_D to m_D .

B Eigenvalues of M_{33}^φ

Let us denote $\mu \equiv m_0 + yv$, $\mu' \equiv y'v'$ and $\mu'' \equiv y''v''$. The moduli of the eigenstates of M_{33}^φ (20) equal

$$\begin{aligned} |M_1| &= |\mu + \mu' + \mu''|, \\ |M_2| &= [|\mu|^2 + |\mu'|^2 + |\mu''|^2 + 2\text{Re}(\omega^2\alpha)]^{1/2}, \\ |M_3| &= [|\mu|^2 + |\mu'|^2 + |\mu''|^2 + 2\text{Re}(\omega\alpha)]^{1/2}. \end{aligned} \quad (85)$$

and $\alpha \equiv \mu\mu'^* + \mu'\mu''^* + \mu''\mu^*$. The arguments of M_2 and M_3 are given by

$$2 \arctan \left[\frac{\text{Im}(2\mu - \mu' - \mu'') \pm \sqrt{3}\text{Re}(\mu' - \mu'')}{|2M_{2,3}| + \text{Re}(2\mu - \mu' - \mu'') \pm \sqrt{3}\text{Im}(\mu'' - \mu')} \right],$$

respectively. If α is real, the moduli of two eigenstates are degenerate.

C Diagonalization of M_{33}^ξ

Consider the diagonalization of the real matrix

$$M_{33}^\xi = \begin{pmatrix} 0 & u_3 & u_2 \\ u_3 & 0 & u_1 \\ u_2 & u_1 & 0 \end{pmatrix}. \quad (86)$$

Its eigenvalues are given by

$$\begin{aligned} m_1 &= \frac{2}{3} \sqrt{3u_1^2 + 3u_2^2 + 3u_3^2} \cdot \cos \frac{\Delta - 2\pi}{3}, \\ m_2 &= \frac{2}{3} \sqrt{3u_1^2 + 3u_2^2 + 3u_3^2} \cdot \cos \frac{\Delta + 2\pi}{3}, \\ m_3 &= \frac{2}{3} \sqrt{3u_1^2 + 3u_2^2 + 3u_3^2} \cdot \cos \frac{\Delta}{3}, \end{aligned} \quad (87)$$

where

$$\cos \Delta \equiv \frac{27u_1u_2u_3}{(3u_1^2 + 3u_2^2 + 3u_3^2)^{3/2}}.$$

If the elements are nearly degenerate as $u_i = u + \epsilon_i$, where $\epsilon_i \ll u$, we have

$$\begin{aligned} m_1 &\simeq -u - \frac{1}{3} \left(\epsilon_1 + \epsilon_2 + \epsilon_3 - 2\sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 - \epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 - \epsilon_3\epsilon_1} \right), \\ m_2 &\simeq -u - \frac{1}{3} \left(\epsilon_1 + \epsilon_2 + \epsilon_3 + 2\sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 - \epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 - \epsilon_3\epsilon_1} \right), \\ m_3 &\simeq 2u + \frac{2}{3} (\epsilon_1 + \epsilon_2 + \epsilon_3). \end{aligned} \quad (88)$$

Under the condition that $|u_3| \leq u_2 \leq u_1$, it is possible to give the three mixing angles [73]:

$$\begin{aligned} \cos^2 \theta_{13} &= \frac{2m_3^2 - u_3^2 - u_2^2}{(m_3 - m_1)(m_3 - m_2)}, \\ \sin^2 \theta_{12} &= \frac{(m_3 - m_1) \cos^2 \theta_{13} - m_3}{(m_2 - m_1) \cos^2 \theta_{13}}, \\ \cos 2\theta_{23} &= \frac{(m_1 - m_2) \sin \theta_{13} \sin 2\theta_{12}}{2u_1}. \end{aligned} \quad (89)$$

Note that the mass matrix always can be redefined to satisfy this condition.

D the TBM and BM mixings

The TBM mixing

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (90)$$

can be generated by the physical neutrino mass matrix in the flavor basis

$$m_\nu = \begin{pmatrix} x+z & y & y \\ y & z & x+y \\ y & x+y & z \end{pmatrix}, \quad (91)$$

whose form is referred to as M_{TBM} in the main text. Here x, y, z are free parameters. How to obtain TBM in other bases have been investigated in sect. 2.5 above.

The BM mixing

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (92)$$

corresponds to the neutrino mass matrix in the flavor basis

$$m_\nu = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}. \quad (93)$$

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